National 5 Mathematics – Question Interpretation Matrix

Unit 1 – Expressions and Formulas

Rationalising Fractions / Rationalising the Denominator	
Look for a fraction with a surd in the denominator	*Example: Solution:* Express $\frac{4}{\sqrt{8}}$ with a rational denominator. Give your answer in its simplest form. Multiply by $\frac{\sqrt{8}}{\sqrt{8}}$ to give $\frac{4}{\sqrt{8}} x \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{8}}{8}$
Factorising Quadratic Expressions	
Three types of quadratic expression: Trinomial: $ax^2 + bx + c$ Difference of Two Squares: $x^2 - a^2$ Common Factor: $x^2 + ax$	*Examples / Solutions:* Trinomial: $x^2 + 8x + 12 = (x + 6)(x + 2)$ Difference of Two Squares: $x^2 - 16 = (x - 4)(x + 4)$ Common Factor: $x^2 + 8x = x(x + 8)$

Completing the Square	
Write $ax^2 + bx + c$ in the form $(x + p)^2 + q$ You just have to learn the process	*Example / Solution:* $x^2 + 6x + 15 = (x + 3)^2 + 6$
Straight Lines not in Standard Form	
Look for a straight line equation which is not in the format $y = mx + c$	*Example / Solution:* $3x - 5y + 15 = 0$ Rearrange to give $y = \frac{3}{5}x + 3$ Read off the gradient, $\frac{3}{5}$, and the y-intercept, 3
Parallel Straight Lines	
Look for questions that mention parallel lines. Parallel lines have the same gradient.	*Example / Solution:* What is the equation of the line which is parallel to the line $y = 3x - 4$ and which passes through the point (1, 5)? Since the lines are parallel they have the same gradient, so $m = 3$. Using $y = mx + c$ gives $y = 3x + 2$

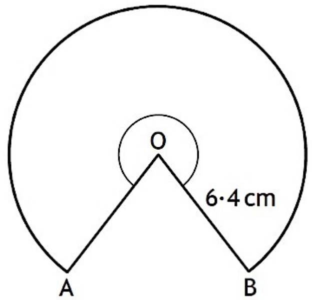
Arc Length / Sector Area Working Backwards

Look for questions that mention 'arc length' or 'sector area' but which give you those values.

Set up the formula as usual and substitute in all of the numbers you can. Use a letter, like a , to represent the angle:

Example / Solution:

The diagram below shows part of a circle, centre O.



The radius of the circle is 6·4 centimetres.

Major arc AB has length 31·5 centimetres.

Calculate the size of the reflex angle AOB.

$$Arc\ Length = \frac{angle}{360°}\ x\ \pi d$$

$$31.5 = \frac{a}{360°}\ x\ \pi\ x\ 12.8$$

$$\frac{31.5\ x\ 360}{12.8\pi} = a$$

Unit 2 – Relationships

Equations with Fractions

Solve by removing the fraction or in the case of two fractions Making a common denominator and then removing the fractions.

Example / Solution:

$$\frac{a-1}{2} = \frac{a+1}{4}$$

$$\frac{2a-21}{4} = \frac{a+1}{4}$$

$$2(a-1) = a+1$$
$$2a-2 = a+1$$
$$a = 3$$

Determining the Nature of the Roots of a Quadratic Equation

Look for the key phrase '<u>nature</u> of the roots' which is asking whether there are no real roots, one real root, or two real roots.

Note that the quadratic expression has to be in the form $ax^2 + bx + c = 0$ before you can determine the $a, b,$ & c values.

For example to process $3x^2 = 2x + 5$ first rearrange to $3x^2 - 2x - 5 = 0$ and then determine the a, b, & c values.

Example / Solution:

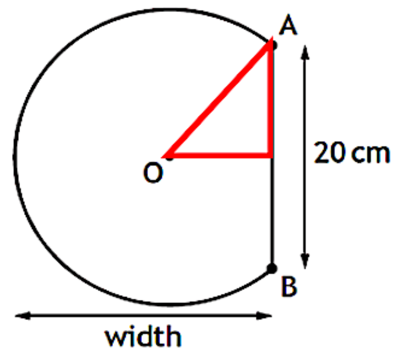
Determine the nature of the roots of $2x^2 - 3x + 1 = 0$

$a = 2, b = -3, c = 1$

$b^2 - 4ac = (-3)^2 - 4(2)(1) = 1$

Since $b^2 - 4ac > 0$ there are two real roots

Determining the Turning Point of a Quadratic Function	
The primary method for finding the turning point is to write the quadratic in completed square form. You can also use the fact that the x co-ordinate of the turning point is at the mid-point of the roots.	*Example / Solution:* Determine the co-ordinates of the turning point of the graph of the quadratic function $y = x^2 - 4x + 5$ $y = x^2 - 4x + 5$ $y = (x-2)^2 + 1$ Turning point is $(2, 1)$
Determining the Y-Intercept of a Quadratic Function	
To determine where the graph of any function crosses over the y-axis let the x co-ordinate = 0 Notice that if the quadratic is written in trinomial format then the y intercept is just the number term i.e. the last term of the quadratic. For example, for $y = x^2 - 4x + 12$ the y intercept is 12 giving the full co-ordinate $(0, 12)$	*Example / Solution:* Determine the co-ordinates of the point where the graph of $y = x^2 - 6x + 7$ intercepts the y-axis Let $x = 0$ to give $y = 0^2 - 6(0) + 7 = 7$ Giving $(0, 7)$

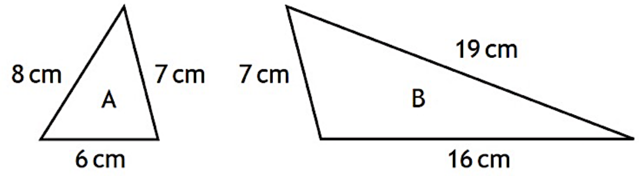
Pythagoras Theorem	
Pythagoras questions will never mention Pythagoras or even present you with a triangle. Look for a circle with a chord line as that is how these questions have always been presented in the past. You'll have to identify where the right-angled triangle needs to go based on what the question is asking you to find.	*Example / Solution:* The shape below is part of a circle, centre O.  The circle has radius 13 centimetres. AB is a chord of length 20 centimetres. Calculate the width of the shape. Let M be the mid-point of AB Construct a right-angled triangle OAM Using Pythagoras, $13^2 - 10^2 = 169 - 100 = 69$ $\sqrt{69} = 8.3$ Width $=$ Radius $+ 8.3 = 13 + 8.3 = 21.3$ cm

Converse of Pythagoras Theorem

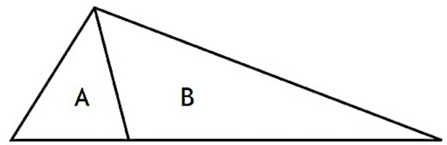
You can identify these questions because they will ask you to show that a triangle is right-angled, although they may not state that explicitly. Look for a triangle that seems right-angled but you haven't been told it's right-angled.

Example / Solution:

Triangles A and B are shown below.



The triangles are placed together to form the larger triangle shown below.



Is this larger triangle right-angled?

Justify your answer.

The hypotenuse of the larger triangle is 22 cm

The short sides have length 8 cm and 19 cm

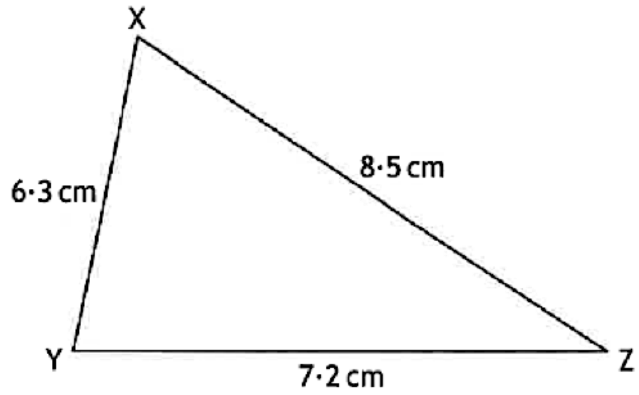
$8^2 + 19^2 = 425$

$22^2 = 484$

Since $425 \neq 484$ the triangle is not right angled by the Converse of Pythagoras

Solving Trigonometric Equations	
Look for an equation which contains one of the trigonometric functions Sin, Cos or Tan.	*Example / Solution:* Solve $11\cos x° - 2 = 3$ for $0 \leq x \leq 360°$ $11\cos x° - 2 = 3$ $11\cos x° = 5$ $\cos x° = \frac{5}{11}$ $x = cos^{-1}\left(\frac{5}{11}\right) = 63°$ Second solution is $360 - 63 = 297°$ from CAST

Unit 3 – Applications

The Sine or Cosine Rule	
Questions involving a triangle can only be Pythagoras or Sine / Cosine Rule. If the triangle is right-angled it’s a Pythagoras question, if not it’s a Sine / Cosine Rule question. It is recommended that for any Sine / Cosine Rule question you draw a fully labelled diagram. To decide whether to use the Sine Rule of the Cosine Rule you have to be familiar with the format of the rules. Spend some time noticing the differences between them.	*Example / Solution:* Triangle XYZ is shown below.  Calculate the size of the smallest angle in triangle XYZ. Since this triangle is not right-angled it has to be a Sine / Cosine question. Since you have all three sides but no angles you need to use the Cosine Rule.

Magnitude of a Vector	
The magnitude of a vector doesn't need interpretation but the formula is often forgotten and it's a question that will almost certainly come up. One tip is that the magnitude of a vector has always been a whole number in past exam questions so if you get a decimal answer it's worth checking your working.	*Example / Solution:* Find the magnitude of the vector $\begin{pmatrix} 6 \\ -13 \\ 18 \end{pmatrix}$ Magnitude $= \sqrt{6^2 + (-13)^2 + 18^2} = 23$ A common mistake is to write (-13^2) which gives -169 which then throws off the final answer
Percentages – Reverse Percentages	
Reverse percentages questions are difficult because the % given in the question is a % of a number you don't know (the number you're actually trying to find in the question). It's tempting to just take the % given of the number given but that's never the correct process.	*Example / Solution:* There are 964 pupils on the roll of Aberleven High School. It is forecast that the roll will decrease by 15% per year. What will be the expected roll after 3 years? Give your answer to the nearest ten. A 15% decrease is the same as 85% of the original roll So, after three years the roll will be given by $964\ x\ 0.85^3 = 592$

Percentages – Appreciation / Depreciation	
Look for questions that refer to a % increase or decrease over a certain timescale, usually in years. You can calculate these ‘year-by-year’ but the process is slow. Instead, use the number of years as a power and use a decimal to represent the % increase or decrease. Remember that a 10% increase is represented by the decimal number 1.1 and a 10% decrease is represented by the number 0.9. Similarly, a 2% increase is represented by 1.02 and a 2% decrease is represented by 0.98.	*Example / Solution:* Households in a city produced a total of 125 000 tonnes of waste in 2017. The total amount of waste is expected to fall by 2% each year. Calculate the total amount of waste these households are expected to produce in 2020. A 2% fall is represented by the decimal 0.98 $125{,}000\ x\ 0.98^3 = 117,649$ $117,649$ tonnes