

National 5 Mathematics

Trigonometry - Solutions - 2014-2019

Marks are indicated in brackets after each question number

2014 Paper 1 Question 10, (2)

The graph has been stretched vertically by a factor of 3 so  $a = 3$

The graph has been moved to the right by  $40^\circ$  so  $b = -40$

2014 Paper 2 Question 12, (3)

$$11\cos x^\circ - 2 = 3$$

$$11\cos x^\circ = 5$$

$$\cos x^\circ = \frac{5}{11}$$

$$x = \cos^{-1}\left(\frac{5}{11}\right) = 63^\circ$$

Second solution is  $360 - 63 = 297^\circ$

2015 Paper 1 Question 6, (2)

Since the amplitude is 4,  $a = 4$

Since there are 3 copies of the Sine graph in  $0 \leq x \leq 360^\circ$ ,  $b = 3$

2015 Paper 1 Question 7, (1) (1) (1)

a) i)  $a = -2$

ii)  $b = -4$

b)  $x = -4$

2015 Paper 1 Question 9, (2)

$$\cos 90^\circ = 0$$

$\cos 100^\circ < 0$  from inspection of the graph

$\cos 300^\circ > 0$  from inspection of the graph

So,  $\cos 100^\circ < \cos 90^\circ < \cos 300^\circ$

2016 Paper 1 Question 11, (2)

$$\tan^2 x \cos^2 x$$

Use  $\tan x = \frac{\sin x}{\cos x}$  to give

$$\left( \frac{\sin x}{\cos x} \right)^2 \cos^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$$

$$\sin^2 x$$

2016 Paper 2 Question 14, (3)

$$2 \tan x + 5 = -4$$

$$2 \tan x = -9$$

$$\tan x = -4.5$$

$$x = \tan^{-1}(-4.5)$$

$$x = 77$$

Using CAST we have

$$x = 180 - 77 = 103^\circ$$

$$x = 360 - 77 = 283^\circ$$

2017 Paper 2 Question 15, (1) (1) (4)

a)  $h = 40 + 23\cos x$

When  $x = 60^\circ$

$$h = 40 + 23\cos 60^\circ$$

$$= 51.5 \text{ m}$$

b) Minimum height occurs where  $x = 180^\circ$

$$h = 40 + 23\cos 180^\circ$$

$$= 17 \text{ m}$$

c) Let  $h = 61$  to give

$$61 = 40 + 23\cos x$$

$$\cos x = \frac{21}{23}$$

$$x = \cos^{-1}\left(\frac{21}{23}\right) = 24^\circ$$

$$x = 360 - 24 = 336^\circ$$

2018 Paper 1 Question 6, (2)

$$y = 5\cos 4x$$

$$a = 5, b = 4$$

2018 Paper 1 Question 10, (3)

$$\begin{aligned}z^2 &= x^2 + y^2 - 2xy\cos Z \\ &= 8^2 + 10^2 - 2(8)(10)\left(\frac{1}{8}\right) \\ &= 164 - 20 \\ &= 144\end{aligned}$$

$$z = 12$$

So,  $XY = 12$  cm

2018 Paper 1 Question 12, (1)

Sketch the graph of  $y = \cos x$

Mark on a horizontal line through 0.5

The line passes through the graph where  $x = 60$

From the symmetry of the graph,  $\cos 240^\circ = -0.5$

Or use a CAST diagram

2018 Paper 1 Question 18, (2)

$\sin x \cos x \tan x$

Substitute  $\tan x = \frac{\sin x}{\cos x}$  to give

$$\sin x \cos x \frac{\sin x}{\cos x}$$

$$= \sin x \sin x$$

$$= \sin^2 x$$

**2018 Paper 2 Question 8, (3)**

$$7\sin x + 2 = 3$$

$$\sin x = \frac{1}{7}$$

$$x = \sin^{-1}\left(\frac{1}{7}\right) = 8.2^\circ$$

From CAST diagram

$$x = 180 - 8.2 = 171.8^\circ$$

**2018 Paper 2 Question 17, (5)**

$$\begin{aligned}\text{Area of Triangle} &= \frac{1}{2}(38)(55)\sin 75 \\ &= 1009.39 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of Sector} &= \frac{75}{360} \times \pi \times 60 \\ &= 39.27 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Shaded Area} &= 1009.39 - 39.27 \\ &= 970.12 \text{ cm}^2\end{aligned}$$

**2019 Paper 1 Question 13, (2)**

$$x\text{-co-ordinate of } A = 180 - 45 = 135$$

$$y\text{-co-ordinate of } A = -1 \times 3 = -3$$

$$\text{Co-ordinates of } A = (135, -3)$$

2019 Paper 2 Question 7, (3)

The smallest angle is at vertex  $Z$

Using the Cosine Rule gives

$$\begin{aligned}\cos Z &= \frac{8.5^2 + 7.2^2 - 6.3^2}{2(8.5)(7.2)} \\ &= \frac{84.4}{122.4}\end{aligned}$$

$$\begin{aligned}Z &= \cos^{-1}\left(\frac{84.4}{122.4}\right) \\ &= 46.4^\circ\end{aligned}$$

2019 Paper 2 Question 14, (3)

$$5\cos x + 2 = 1$$

$$5\cos x = -1$$

$$\cos x = -\frac{1}{5}$$

$$\cos x = -0.2$$

$$\cos^{-1}(0.2) = 78^\circ$$

From CAST solutions lie in quadrants 2 & 3, giving

$$x = 180 - 78 = 102^\circ$$

$$x = 180 + 78 = 258^\circ$$

2019 Paper 2 Question 17, (2)

$$(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$$

$$= \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$= \sin^2 x + \cos^2 x + 2\sin x \cos x$$

$$[\sin^2 x + \cos^2 x = 1 \text{ from Trig Identities}]$$

$$= 2\sin x \cos x + 1$$