Higher Mathematics

Trigonometric Equations - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 15, (2)

$\tan\frac{\pi}{4} = 1$ so $\frac{\pi}{4}$ is our reference angle

Using CAST tan is negative in quadrants 2 & 4

So, the solutions are $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$ and $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

However, these solutions are for $\frac{x}{2}$ giving $x = \frac{3\pi}{2},\ x = \frac{5\pi}{2}$

But $\frac{5\pi}{2}$ is out of our range so $x = \frac{3\pi}{2}$

2013 Paper 2 Question 8, (6)

$\sin 2x = 2\cos^2 x$

$2\sin x\cos x = 2\cos^2 x$

$2\sin x\cos x - 2\cos^2 x = 0$

$2\cos x\left(\sin x - \cos x\right) = 0$

Separating into two equations gives

$2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2},\ \frac{3\pi}{2}$

$\sin x - \cos x = 0$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4},\ \frac{5\pi}{4}$

2014 Paper 2 Question 6, (5)

$\sin x - 2\cos 2x = 1$

Using the double angle formula $\cos 2x = 1 - 2\sin^2 x$ we have

$\sin x - 2\left(1 - 2\sin^2 x\right) = 1$

$\sin x - 2 + 4\sin^2 x = 1$

$4\sin^2 x + \sin x - 3 = 0$

$(4\sin x - 3)(\sin x + 1) = 0$

Separating into two equations gives

$4\sin x - 3 = 0$ $\quad$ $\sin x + 1 = 0$

$\sin x = \frac{3}{4}$ $\quad$ $\sin x = -1$

$x = 0.8\ rad$ $\quad$ $x = \frac{3\pi}{2}$

$x = \pi - 0.8 = 2.3\ rad$

2017 Paper 2 Question 6, (5)

$5\sin x - 4 = 2\cos 2x$

$5\sin x - 4 = 2(1 - 2\sin^2 x)$ from double angle formula

$5\sin x - 4 = 2 - 4\sin^2 x$

Rearranging gives

$4\sin^2 x + 5\sin x - 6 = 0$

$(4\sin x - 3)(\sin x + 2) = 0$

Separating into two equations gives

$4\sin x - 3 = 0$ and $\sin x + 2 = 0$

$4\sin x - 3 = 0$

$\sin x = \frac{3}{4}$

$x = sin^{-1}\left(\frac{3}{4}\right) = 0.85$ radians

$x = \pi - 0.85 = 2.3$ radians

$\sin x + 2 = 0$

$\sin x = -2$

No solutions since $-1 \leq \sin x \leq 1$

<u>2018 Paper 2 Question 6, (3)</u>

a) i) $f\big(g(x)\big) = 3 + \cos(2x)$

ii) $g\big(f(x)\big) = 2(3 + \cos x)$

$= 6 + 2\cos x$

b) $3 + \cos(2x) = 6 + 2\cos x$

$\cos(2x) - 2\cos x - 3 = 0$

$2cos^2x - 1 - 2\cos x - 3 = 0$

$2cos^2x - 2\cos x - 4 = 0$

$cos^2x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0$

$\cos x = 2$

No solutions since $-1 \leq \cos x \leq 1$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

<u>2019 Paper 1 Question 15, (4) (1)</u>

a) $\sin 2x + 6\cos x = 0$

$2\sin x\cos x + 6\cos x = 0$

$2\cos x\,(\sin x + 3) = 0$

$2\cos x = 0$

$\cos x = 0$

$x = 90°,\ 270°$

$\sin x + 3 = 0$

$\sin x = -3$

No solutions since $-1 \leq \sin x \leq 1$

b) $\sin 4x + 6\cos 2x = 0$

Compare with the equation in a) to see that the angles are doubled meaning that the graph of this function is compressed by a factor of 2. This means that the solutions to b) are half those of a) giving $x = 45°,\ 135°$

Also, additional solutions are introduced because of the period change of the function in b). These are found by adding $360°$ to the solutions from a) and then dividing by 2.

$x = \dfrac{90 + 360}{2} = 225°$ and $x = \dfrac{270 + 360}{2} = 315°$