Higher Mathematics

Straight Lines - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 5, (2)

$5x + 3y - 6 = 0$

Rearranging gives

$y = -\frac{5}{3}x + 2$

Gradient of this line is $-\frac{5}{3}$

Since L is parallel it also has gradient $-\frac{5}{3}$

Using $y - b = m(x - a)$ with (-2, -1) gives

$y - (-1) = -\frac{5}{3}(x - (-2))$

Rearranging gives

$y = -\frac{5}{3}x - \frac{8}{3}$

2013 Paper 2 Question 2, (3) (3) (3)

a) $m_{PQ} = \frac{6-2}{5-7} = -2$

$m_{QR} = \frac{1}{2}$ since $PQ \;\; QR$ are perpendicular

Using $y - b = m(x - a)$ with $Q = (5, 6)$ we have

$y - 6 = \frac{1}{2}(x - 5)$

$y = \frac{1}{2}x + \frac{7}{2}$

b) $x + 3y = 13$

$y = -\frac{1}{3}x + \frac{13}{3}$

Equating QR with PT gives

$\frac{1}{2}x + \frac{7}{2} = -\frac{1}{3}x + \frac{13}{3}$

Multiply through by 6 to simplify

$3x + 21 = -2x + 26$

$x = 1$

Substituting $x = 1$ into $y = \frac{1}{2}x + \frac{7}{2}$ gives

$y = \frac{1}{2} + \frac{7}{2} = 4$

$T = (1, 4)$

c) $\overrightarrow{QT} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

So, $R = (1 - 4, 4 - 2) = (-3, 2)$

$\overrightarrow{QP} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

So, $S = (-3 + 2, 2 - 4) = (-1, -2)$

<u>2014 Paper 1 Question 2, (2)</u>

$m_{CT} = \frac{2 - (-1)}{1 - 3} = -\frac{3}{2}$

$m_{tan} = \frac{2}{3}$ since perpendicular

Using $y - b = m(x - a)$ with $(3, -1)$ gives

$y - (-1) = \frac{2}{3}(x - 3)$

$y = \frac{2}{3}x - 3$

2014 Paper 2 Question 1, (4) (2) (2)

$a)\ m_{AB} = \frac{2-0}{5-3} = 1$

So, gradient of perpendicular line is -1

Midpoint of AB $= (4, 1)$

Using $y - b = m(x - a)$ with (4, 1) we have

$y - 1 = -1(x - 4)$

$y = -x + 5$ (1)

b) $y + 2x = 6$

$y = -2x + 6$ (2)

Equating (1) & (2) gives

$-x + 5 = -2x + 6$

$x = 1$

When $x = 1, y = 4$

So, $T = (1, 4)$

c) $A = (3, 0),\ T = (1, 4)$

$m_{AT} = \frac{4-0}{1-3} = -2$

Using $y - b = m(x - a)$ with (1, 4) gives

$y - 4 = -2(x - 1)$

Rearranging gives

$y = -2x - 2$

2015 Paper 1 Question 9, (3)

$y + \sqrt{3}x = 0$

$y = -\sqrt{3}x$

So, AB has a gradient of $-\sqrt{3}$ since parallel

$\tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}$

So, BC has a gradient of $-\frac{\sqrt{3}}{3}$

Thus A,B, C cannot be collinear

<u>2015 Paper 2 Question 1, (4) (3)</u>

a) $m_{AB} = \frac{7-(-5)}{-5-(-1)} = -3$

$m_{alt} = \frac{1}{3}$ since perpendicular to AB

Using $y - b = m(x - a)$ with $(13, 3)$ gives

$y - 3 = \frac{1}{3}(x - 13)$

Rearranging gives

$y = \frac{1}{3}x + \frac{26}{3}$

b) Mid-point of AC $= (4, 5)$ by inspection

$m_{med} = \frac{5-(-5)}{4-(-1)} = 2$

Using $y - b = m(x - a)$ with $(4, 5)$ we have

$y - 5 = 2(x - 4)$

$y = 2x - 3$

c) Equating the two lines gives

$\frac{1}{3}x + \frac{26}{3} = 2x - 3$

$x + 26 = 6x - 9$

$5x = 35$

$x = 7$

When $x = 7$, $y = 11$

So, the point of intersection is $(7, 11)$

2016 Paper 1 Question 1, (2)

$y + 4x = 7$

$y = -4x + 7$

Any line parallel to this has gradient of -4

Using $y - b = m(x - a)$ with $(-2, 3)$ gives

$y - 3 = -4(x - (-2)$

$y - 3 = -4(x + 2)$

$y = -4x - 5$

2016 Paper 2 Question 1, (1) (2) (3)

a) i) $M = (2, 4)$

ii) $m_{PM} = \dfrac{-4 - 4}{0 - 2} = 4$

Using $y - b = m(x - a)$ with (2, 4) gives

$y - 4 = 4(x - 2)$

$y = 4x - 4$

b) $m_{PR} = \dfrac{6 - (-4)}{10 - 0} = 1$

So, $m_{perp} = -1$

Using $y - b = m(x - a)$ with (2, 4) gives

$y - 4 = -1(x - 2)$

$y = -x + 6$

c) Midpoint of PR $= (5, 1)$

Test whether (5, 1) lies on L

Substituting $x = 5$ into L gives

$y = -5 + 6 = 1$

So, (5, 1) does lie on the line L

2017 Paper 1 Question 7, (3)

Midpoint of AB $= (2, 7)$

Let M $= (2, 7)$

$m_{CM} = \dfrac{11-7}{2-2} =$ undefined

So, the line is vertical and since it goes through (2, 7) has equation

$x = 2$

2017 Paper 1 Question 11, (3)

$3y - 2x = 4$

$y = \dfrac{2}{3}x + \dfrac{4}{3}$

The gradient of this line is $\dfrac{2}{3}$

The gradient of the line joining A&B is also $\dfrac{2}{3}$ since parallel

$m_{AB} = \dfrac{2-a}{-7-5} = \dfrac{2}{3}$

$\dfrac{2-a}{-12} = \dfrac{2}{3}$

$2 - a = -8$

$a = 10$

2017 Paper 2 Question 1, (4) (2) (2)

a) $m_{BC} = \dfrac{0-(-2)}{3-9} = -\dfrac{1}{3}$

$m_{perp} = 3$

Midpoint of BC $= (6, -1)$

Using $y - b = m(x - a)$ with $(6, -1)$ gives

$y - (-1) = 3(x - 1)$

$y = 3x - 19$

b) $m_{AB} = \tan 45° = 1$

Using $y - b = m(x - a)$ with $(3, 0)$ gives

$y - 0 = (x - 3)$

$y = x - 3$

c) Equating gives $3x - 19 = x - 3$

$2x = 16$

$x = 8$

When $x = 8$, $y = 5$

So, the point of intersection is (8, 50)

<u>2018 Paper 1 Question 1, (3)</u>

Mid-point of PQ $= (1, 2)$

Let $s = (1, 2)$

$$m_{RS} = \frac{6 - 2}{3 - 1} = 2$$

Using $y - b = m(x - a)$ with $(1, 2)$ gives

$y - 2 = 2(x - 1)$

$y - 2 = 2x - 2$

$y = 2x$

<u>2018 Paper 1 Question 5, (1) (1)</u>

a) $\frac{8}{10} = \frac{4}{5}$

b) $\frac{4}{5}$ of $(9 - 4) = \frac{4}{5}\, x\, 5$

$= 4$

So, $t = 4$

2018 Paper 1 Question 8, (2)

$y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

Gradient $= \sqrt{3}$

So, $\tan\theta = \sqrt{3}$

$\theta = 60°$ from exact values

2018 Paper 2 Question 5, (3) (2)

a) Mid-point $PQ = (6, 1)$

$$m_{PQ} = \frac{4 + 2}{3 - 9} = \frac{6}{-6} = -1$$

So, $m_{L1} = 1$

$y - b = m(x - a)$

$y - 1 = x - 6$

$y = x - 5$

b) $3y + x = 25$

$3y = 25 - x$

$y = x - 5$

$3y = 3x - 15$

Equating gives

$3x - 15 = 25 - x$

$4x = 40$

$x = 10$

When $x = 10, y = 10 - 5 = 5$

So, $C = (10, 5)$

2019 Paper 1 Question 7, (4)

$\tan 30° = m = \dfrac{1}{\sqrt{3}}$

Using $y - b = m(x - a)$ gives

$$y + 4 = \frac{1}{\sqrt{3}}(x - 0)$$

$$y + 4 = \frac{1}{\sqrt{3}}x$$

$$y = \frac{1}{\sqrt{3}}x - 4$$

2019 Paper 2 Question 1, (3) (3) (2)

a) Since D is the mid-point of AC it has co-ordinates $(-4, -3)$

$$m_{BD} = \frac{-3+8}{-4-11} = -\frac{1}{3}$$

Using $y - b = m(x - a)$ with $(11, -8)$ gives

$$y + 8 = -\frac{1}{3}(x - 11)$$

$$3y + 24 = -x + 11$$

$3y = -x - 13$ (1)

b) $m_{BC} = \frac{6 + 8}{-3 - 11} = \frac{14}{-14} = -1$

$m_{AE} = 1$ since $AE\ \ BC$ are perpendicular $\rightarrow m_{BC} \bullet m_{AE} = -1$

Using $y - b = m(x - a)$ with $(-5, -12)$ gives

$y + 12 = x + 5$

$y = x - 7$ (2)

c) By substituting (2) into (1) we have

$3(x - 7) = -x - 13$

$3x - 21 = -x - 13$

$4x = 8$

$x = 2$

When $x = 2$, $y = 2 - 7 = -5$ using equation (2)

So, the point of intersection is $(2, -5)$