Higher Mathematics

Recurrence Relations - **Questions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 8, (2)

A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

Here are two statements about this sequence:

(1) $u_0 = 9{\cdot}1$;

(2) The sequence has a limit as $n \longrightarrow \infty$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

2013 Paper 2 Question 1, (4)

The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of m and c.

2014 Paper 1 Question 1, (2)

A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.

What is the value of u_4?

2014 Paper 1 Question 10, (2)

A sequence is defined by the recurrence relation

$u_{n+1} = (k-2)u_n + 5$ with $u_0 = 3$.

For what values of k does this sequence have a limit as $n \to \infty$?

2015 Paper 2 Question 3, (1) (5)

A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32,$ $\qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13,$ $\qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day.

(b) Determine whether or not either of them will eventually escape from the well.

2016 Paper 1 Question 3, (1) (1) (2)

A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4.

(b) Explain why this sequence approaches a limit as $n \to \infty$.

(c) Calculate this limit.

2017 Paper 1 Question 9, (2) (1)

A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m.

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$.

(ii) Calculate this limit.

2017 Paper 2 Question 8, (2) (4)

Sequences may be generated by recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$.

(b) Determine the range of values of k for which $u_2 < u_0$.

2018 Paper 2 Question 7, (2) (2) (1) (3) (1)

(a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$.

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully.

The fifth term, u_5, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$.

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of a.

(ii) Calculate the limit.

2019 Paper 1 Question 4, (3) (1)

A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9 and 11.

(a) Find the values of m and c.

(b) Hence, calculate the fourth term of the sequence.

2019 Paper 2 Question 4, (1) (1) (2)

In a forest, the population of a species of mouse is falling by 2·7% each year.

To increase the population scientists plan to release 30 mice into the forest at the end of March each year.

(a) u_n is the estimated population of mice at the start of April, n years after the population was first estimated.

It is known that u_n and u_{n+1} satisfy the recurrence relation $u_{n+1} = au_n + b$.

State the values of a and b.

The scientists continue to release this species of mouse each year.

(b) (i) Explain why the estimated population of mice will stabilise in the long term.

(ii) Calculate the long term population to the nearest hundred.