Higher Mathematics

Rates of Change - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2014 Paper 2 Question 9, (3) (2) (3)

a) $v(t) = 8\cos(2t - \frac{\pi}{2})$

$a(t) = v'(t) = -16\sin(2t - \frac{\pi}{2})$

b) $v'(10) = -16\sin\left(20 - \frac{\pi}{2}\right) = 6.53$

Since $6.53 > 0$ the velocity of the object is increasing when $t = 10$

c) $s(t) = \int v(t)\, dt$

$= \int 8\cos(2t - \frac{\pi}{2})dt$

$= 4\sin\left(2t - \frac{\pi}{2}\right) + c$

Since $s(t) = 4$ when $t = 0$ we have

$4 = 4\sin(0 - \frac{\pi}{2}) + c$

$4 = 4\sin\left(-\frac{\pi}{2}\right) + c$

$4 = 4 \bullet (-1) + c$

$c = 8$

So, $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$

2015 Paper 1 Question 15, (6)

$$\frac{dT}{dt} = \frac{1}{25}t - k$$

$$T(t) = \int \frac{dT}{dt}\, dt$$

$$= \int (\frac{1}{25}t - k)\, dt$$

$$= \frac{1}{50}t^2 - kt + c$$

Now we must find *k* & *c* using the information given in the question

At $t = 0, T = 100$ (note that 'initially' means $t = 0$)

So, $100 = \left(\frac{1}{50} \bullet 0\right) - \left(k \bullet 0\right) + c$

$c = 100$

At $t = 10, T = 82$

So, $82 = \left(\frac{1}{50} \bullet 10^2\right) - 10k + 100$

$$82 = 2 - 10k + 100$$

$$10k = 20$$

$$k = 2$$

So, $T(t) = \frac{1}{50}t^2 - 2t + 100$

2017 Paper 1 Question 8, (3)

$$d(t) = \frac{1}{2t} = \frac{t^{-1}}{2} = \frac{1}{2}t^{-1}$$

$$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$$

$$d'(5) = -\frac{1}{2 \bullet 5^2} = -\frac{1}{50}$$

So, the rate of change of $d(t)$ when $t = 5$ is $-\frac{1}{50}$