Higher Mathematics

Quadratics - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 3, (2)

$a = 2, b = 4, c = 5$

$b^2 - 4ac = 4^2 - 4 \bullet 2 \bullet 5$

$= 16 - 40$

$= -24$

2013 Paper 1 Question 19, (2)

$1 - 2x - 3x^2 > 0$

Multiplying through by -1 gives

$3x^2 + 2x - 1 < 0$

$(3x - 1)(x + 1) < 0$

Consider $(3x - 1)(x + 1) = 0$

$x = -1, x = \frac{1}{3}$

These are the roots of the quadratic. By considering the graph we see

It is negative (i.e. $y < 0$) for $-1 < x < \frac{1}{3}$

The solutions are all x such that $-1 < x < \frac{1}{3}$

2013 Paper 1 Question 21, (3)

$2x^2 + 12x + 1$

$= 2\left(x^2 + 6x\right) + 1$

Now complete the square on the inside of the bracket

$= 2\left[(x + 3)^2 - 9\right] + 1$

$= 2(x+3)^2 - 18 + 1$

$= 2(x+3)^2 - 17$

<u>2014 Paper 1 Question 17, (2)</u>

$$\begin{aligned} 3x^2 + 12x + 17 &= 3\left(x^2 + 4x\right) + 17 \\ &= 3\left[(x+2)^2 - 4\right] + 17 \\ &= 3(x+2)^2 - 12 + 17 \\ &= 3(x+2)^2 + 5 \end{aligned}$$

<u>2015 Paper 1 Question 8, (4)</u>

Area $= x(x-2) < 15$

$x^2 - 2x - 15 < 0$

$(x-5)(x+3) < 0$

Consider $\left(x-5\right)(x+3) = 0$

$x = -3, x = 5$

Since the graph of $x^2 - 2x - 15$ is U-shaped with roots at $x = -3, x = 5$ we have

$(x-5)(x+3) < 0$ for $-3 < x < 5$

<u>2015 Paper 1 Question 11, (4) (6)</u>

a) Circle centre $= (-8, -2)$, radius $= \sqrt{45}$

Let circle centre be C

$$m_{CT} = \frac{-2-(-5)}{-8-(-2)} = -\frac{1}{2}$$

$m_{tan} = 2$ since the tangent is perpendicular to CT

Using $y - b = m(x - a)$ with $(-2, -5)$ gives

$y - (-5) = 2(x - (-2))$

$y + 5 = 2x + 4$

$y = 2x - 1$

b) Consider where the line, $y = 2x - 1$, and the parabola meet

To find this point equate them to give

$2x - 1 = -2x^2 + px + 1 - p$

$2x^2 + 2x - px + p - 2 = 0$

Grouping terms gives

$2x^2 + (2 - p)x + (p - 2) = 0$

The solutions to this equation are the points where the line and parabola meet.

Since the line is a tangent to the parabola they meet at only one point.

$a = 2, b = (2 - p), \ c = (p - 2)$

Since there is one solution $b^2 - 4ac = 0$

$$b^2 - 4ac = (2 - p)^2 - 4 \bullet 2 \bullet (p - 2) = 0$$

$$4 - 4p + p^2 - 8p + 16 = 0$$

$$p^2 - 12p + 20 = 0$$

$$(p - 10)(p - 2) = 0$$

$$p = 2, p = 10$$

But since $p > 3$ the solution is $p > 10$

<u>2016 Paper 1 Question 12, (2) (3)</u>

a) $h(x) = f(g(x)) = f(3 - x) = 2(3 - x)^2 - 4(3 - x) + 5 = 2x^2 - 8x + 11$

b) $h(x) = 2x^2 - 8x + 11 = 2(x^2 - 4x) + 11$

$$= 2\left[(x - 2)^2 - 4\right] + 11$$

$$= 2(x - 2)^2 - 8 + 11$$

$$= 2(x - 2)^2 + 3$$

2016 Paper 2 Question 2, (3)

$x^2 - 2x + 3 - p = 0$

$a = 1, b = -2, c = 3 - p$

For no real roots $b^2 - 4ac < 0$

$b^2 - 4ac < 0$

$(-2)^2 - 4 \bullet 1 \bullet (3 - p) < 0$

$4 - 12 + 4p < 0$

$4p < 8$

$p < 2$

2017 Paper 1 Question 4, (3)

$x^2 + 4x + \left(k - 5\right) = 0$

$a = 1, b = 4, c = k - 5$

For equal roots $b^2 - 4ac = 0$

$4^2 - 4 \bullet 1 \bullet \left(k - 5\right) = 0$

$16 - 4k + 20 = 0$

$4k = 36, k = 9$

2017 Paper 2 Question 4, (3) (2) (2)

a) $3x^2 + 24x + 50 = 3\left(x^2 + 8x\right) + 50$

$= 3\left[(x + 4)^2 - 16\right] + 50$

$= 3(x + 4)^2 - 48 + 50$

$= 3(x + 4)^2 + 2$

b) $f(x) = x^3 + 12x^2 + 50x - 11$

$f'(x) = 3x^2 + 24x + 50$

c) $f'(x) = 3x^2 + 24x + 50$

$= 3(x + 4)^2 + 2$ (from a)

Since $(x+4)^2 \geq 0, 3(x+4)^2 + 2 > 0$

So, $f'(x) > 0$ meaning that $f(x)$ is strictly increasing for all x

2018 Paper 2 Question 4, (3)

$$-3x^2 - 6x + 7 = -3\left(x^2 + 2x\right) + 7$$

$$= -3\left[(x+1)^2 - 1\right] + 7$$

$$= -3(x+1)^2 + 3 + 7$$

$$= -3(x+1)^2 + 10$$

2018 Paper 2 Question 10, (4)

$x^2 + (m-3)x + m = 0$

$a = 1, b = m - 3, c = m$

$b^2 - 4ac = (m-3)^2 - 4(1)(m)$

$= m^2 - 6m + 9 - 4m$

$= m^2 - 10m + 9$

Since there are two roots, $b^2 - 4ac > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

Consider the roots of $m^2 - 10m + 9$ which are at $m = 1, m = 9$

Consider the graph of $m^2 - 10m + 9$ giving

$m < 1, m > 9$

2019 Paper 1 Question 2, (3)

$x^2 + (k-5)x + 1 = 0$

For equal roots $b^2 - 4ac = 0$

$a = 1, b = k - 5, c = 1$

$b^2 - 4ac = (k-5)^2 - 4(1)(1)$

$= k^2 - 10k + 25 - 4$

$= k^2 - 10k + 21$

$k^2 - 10k + 21 = 0$

$(k-3)(k-7) = 0$

$k = 3, k = 7$

2019 Paper 2 Question 7, (3)

a) $-6x^2 + 24x - 25 = -6(x^2 - 4x) - 25$

$= -6\left[(x-2)^2 - 4\right] - 25$

$= -6(x-2)^2 + 24 - 25$

$= -6(x-2)^2 - 1$