Higher Mathematics

Polynomials - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 6, (2)

Let $f(x) = x^3 + 3x^2 - 5x - 6$

$f(2) = 2^3 + 3 \bullet 2^2 - 5 \bullet 2 - 6 = 4$

The remainder is 4 when $f(x)$ is divided by $(x - 2)$

2013 Paper 1 Question 17, (2)

$y = kx(x + a)^2$

Rewriting this gives

$y = k(x - 0)(x + a)(x + a)$

This shows that the roots are at $x = 0, x = -a$

So, by inspection, $a = 2$

So $y = kx(x + 2)^2$

Substituting the point (1, 3) gives

$3 = k(1 + 2)^2$

$3 = 9k$

$k = \frac{1}{3}$

2013 Paper 2 Question 3, (4) (5)

a) Let $f(x) = x^3 + 3x^2 + x - 5$

Using synthetic division gives

1	1	3	1	-5
		1	4	5
	1	4	5	0

$f(x) = (x-1)(x^2+4x+5)$

Note that x^2+4x+5 does not factorise

b) Let $g(x) = x^4+4x^3+2x^2-20x+3$

$g'(x) = 4x^3+12x^2+4x-20$

Stationary Points occur where $g'(x) = 0$ giving

$4x^3+12x^2+4x-20 = 0$

$x^3+3x^2+x-5 = 0$

$(x-1)\left(x^2+4x+5\right) = 0$ (from a)

The solution to this equation is $x = 1$ since x^2+4x+5 has no solutions

$(b^2-4ac = -4 < 0)$

Thus $g(x)$ has only one stationary point

2014 Paper 1 Question 15, (2)

Since the roots are at $x = -1, x = 2$ we have $y = k(x+1)(x-2)^2$ for some value k

Substituting the point $(0, -8)$ gives

$-8 = k(0+1)(0-2)^2$

$-8 = 4k$

$k = -2$

So, $y = -2(x+1)(x-2)^2$

2014 Paper 1 Question 22, (4) (3)

a) Let $f(x) = 6x^3+7x^2+ax+b$

Since $x+1$ is a factor $f(-1) = 0$

$f(-1) = -6+7-a+b = 0$

Rearranging gives $a = b+1$ (1)

Since 72 is the remainder when $f(x)$ is divided by $x-2$ we have

$f(2) = 48+28+2a+b = 72$

Rearranging gives $2a = -b-4$ (2)

Substituting (1) into (2) gives

$$2(b+1) = -b-4$$

$$2b+2 = -b-4$$

$$3b = -6$$

$$b = -2$$

Using (1) we have

$$a = -2+1 = -1$$

b) $f(x) = 6x^3 + 7x^2 - x - 2$

Since $x+1$ is a factor we have

-1	6	7	-1	-2
		6	-1	2
	6	1	-2	0

So, $f(x) = (x+1)(6x^2 + x - 2)$

$= (x+1)(3x+2)(2x-1)$

2015 Paper 1 Question 3, (4)

Let $f(x) = x^3 - 3x^2 - 10x + 24$

Using synthetic division gives

-3	1	-3	-10	24
		-3	18	-24
	1	-6	8	0

Since the remainder is zero, $(x+3)$ is a factor of $f(x)$

$f(x) = (x+3)(x^2 - 6x + 8)$

$= (x+3)(x-4)(x-2)$

2016 Paper 1 Question 15, (3) (1)

a) $f(x) = k(x-a)(x-b)^2$

By inspection of the roots we have

$f(x) = k(x-4)(x+5)^2$

Substituting the point (1, 9) gives

$9 = k(1-4)(1+5)^2$

Rearranging and solving for k gives

$k = -\frac{1}{12}$

b) $g(x) = f(x) - d$

Since d is positive the graph of $g(x)$ is the graph of $f(x)$ moved down by $'d'$ units

By considering the graph of $f(x)$ it must be moved down by at least 9 units to give only one root – in other words to move the point (1, 9) below the x axis.

So, $d > 9$

2016 Paper 2 Question 3, (2) (3) (1) (4)

a) i) Let $f(x) = 2x^3 - 9x^2 + 3x + 14$

Using synthetic division gives

-1	2	-9	3	14
		-2	11	-14
	2	-11	14	0

Since the remainder is zero $(x+1)$ is a factor of $f(x)$

ii) $2x^3 - 9x^2 + 3x + 14 = 0$

$(x+1)\left(2x^2 - 11x + 14\right) = 0$

$(x+1)(2x-7)(x-2) = 0$

$x = -1, x = 2, x = \frac{7}{2}$

b) i) $y = 2x^3 - 9x^2 + 3x + 14$

$y = (x + 1)(2x - 7)(x - 2)$

By inspection of the roots $A = (-1, 0),\ B = (2, 0)$

ii) Area $= \int_{-1}^{2} \left(2x^3 - 9x^2 + 3x + 14\right) dx$

$= [\frac{2}{4}x^4 - \frac{9}{3}x^3 + \frac{3}{2}x^2 + 14x]_{-1}^{2}$

$= [\frac{1}{2}x^4 - 3x^3 + \frac{3}{2}x^2 + 14x]_{-1}^{2}$

$= \left(8 - 24 + 6 + 28\right) - (\frac{1}{2} + 3 + \frac{3}{2} - 14)$

$= 27$

Area $= 27\ units^2$

<u>2017 Paper 2 Question 2, (2) (3)</u>

a) $f(x) = 2x^3 - 5x^2 + x + 2$

Using synthetic division gives

1	2	-5	1	2
		2	-3	-2
	2	-3	-2	0

Since the remainder is zero, $x - 1$ is a factor of $f(x)$

b) $f(x) = 2x^3 - 5x^2 + x + 2$

$= (x - 1)(2x^2 - 3x - 2)$

$= (x - 1)(2x + 1)(x - 2)$

For $f(x) = 0$ we have

$(x - 1)(2x + 1)(x - 2) = 0$

$x = -\frac{1}{2}$, $x = 1, x = 2$

2018 Paper 1 Question 7, (1) (3) (4)

a) $(0, 5)$

b) $y = x^3 - 3x^2 + 2x + 5$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$m_{tan} = 3(0)^2 - 6(0) + 2 = 2$

$y = 2x + 5$

c) $2x + 5 = x^3 - 3x^2 + 2x + 5$

$x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

$x = 0, x = 3$

When $x = 3$, $y = (2 x 3) + 5$

$= 11$

$Q = (3, 11)$

2018 Paper 1 Question 15, (5)

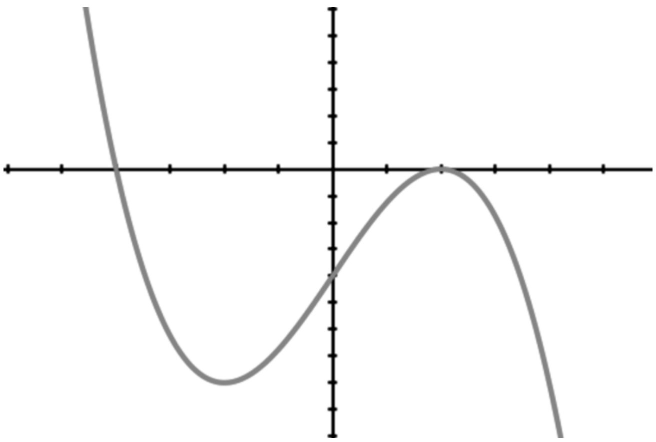
Since $(x + 4)$ is a factor, $x = -4$ is a root

Since $x = 2$ is a repeated root, $(x - 2)$ is a repeated factor

So, the equation of the function can be written $f(x) = k(x + 4)(x - 2)(x - 2)$

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$

Considering all of the above information and assuming a k value (which cannot be determined from the information) a possible graph is:



2018 Paper 2 Question 7, (2) (2)

a) i)

2	2	-3	-3	2
		4	2	-2
	2	1	-1	0

Zero remainder shows that $(x-2)$ is a factor

ii) $2x^3 - 3x^2 - 3x + 2 = (x-2)(2x^2 + x - 1)$

$$= (x-2)(2x-1)(x+1)$$

2019 Paper 2 Question 10, (2) (5)

a)

- 3	3	10	1	- 8	- 6
		- 9	- 3	6	6
	3	1	- 2	- 2	0

Since the remainder is 0, $(x+3)$ is a factor

b) $3x^4 + 10x^3 + x^2 - 8x - 6$

$= (x+3)(3x^3 + x^2 - 2x - 2)$

We need to factorise $3x^3 + x^2 - 2x - 2$. Let $g(x) = 3x^3 + x^2 - 2x - 2$

By inspection, $(x-1)$ is a factor since $g(1) = 0$

1	3	1	- 2	-2
		3	4	2
	3	4	2	0

So, $3x^4 + 10x^3 + x^2 - 8x - 6 = (x+3)(x-1)(3x^2 + 4x + 2)$

Note this does not factorise any further