Higher Mathematics

Optimization - **Questions** - 2013-2019

Marks are indicated in brackets after each question number

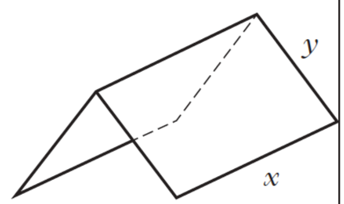
2013 Paper 2 Question 7, (3) (7)

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,\text{m}^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

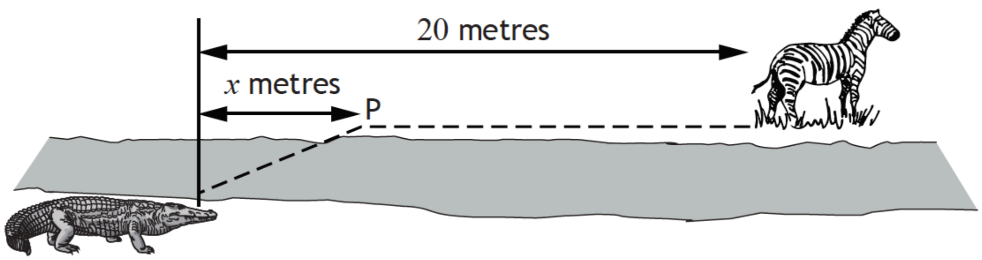
(ii) Calculate the minimum cost of a frame.

<u>2015 Paper 2 Question 8, (1) (1) (8)</u>

A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land.

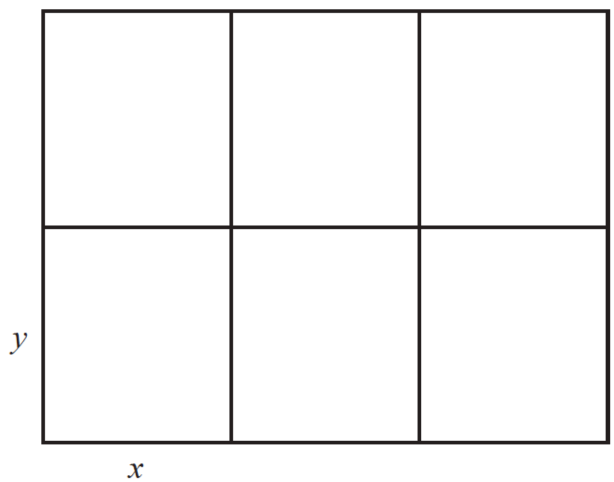
(ii) Calculate the time taken if the crocodile swims the shortest distance possible.

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time.

2016 Paper 2 Question 7, (3) (6)

A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is $108\ \text{m}^2$.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$

(b) Find the value of x that minimises the length of fencing required.

2018 Paper 2 Question 9, (6)

A sector with a particular fixed area has radius x cm.

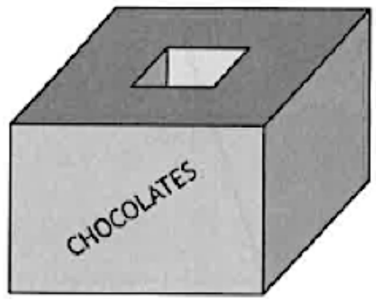
The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P.

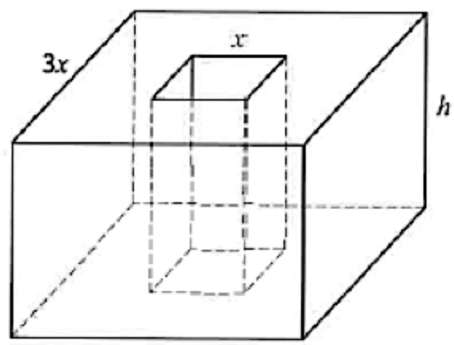
2019 Paper 2 Question 11, (3) (6)

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is $2000\,\text{cm}^3$



(a) Show that the total surface area, A cm^2, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

Find the minimum value of A.