

Higher Mathematics

Inverse Functions - Solutions - 2013-2019

Marks are indicated in brackets after each question number

2015 Paper 1 Question 5, (2) (1)

a) $g(x) = 6 - 2x$

$$y = 6 - 2x$$

Interchange x & y to give

$$x = 6 - 2y$$

Solve for y

$$y = 3 - \frac{1}{2}x$$

$$\text{So, } g^{-1}(x) = 3 - \frac{1}{2}x$$

b) $g(g^{-1}(x)) = x$ since g and g^{-1} are inverses of each other

2016 Paper 1 Question 6, (3) (1)

a) $f(x) = 3x + 5$

Let $y = 3x + 5$

Swap x & y to give

$$x = 3y + 5$$

$$y = \frac{x - 5}{3}$$

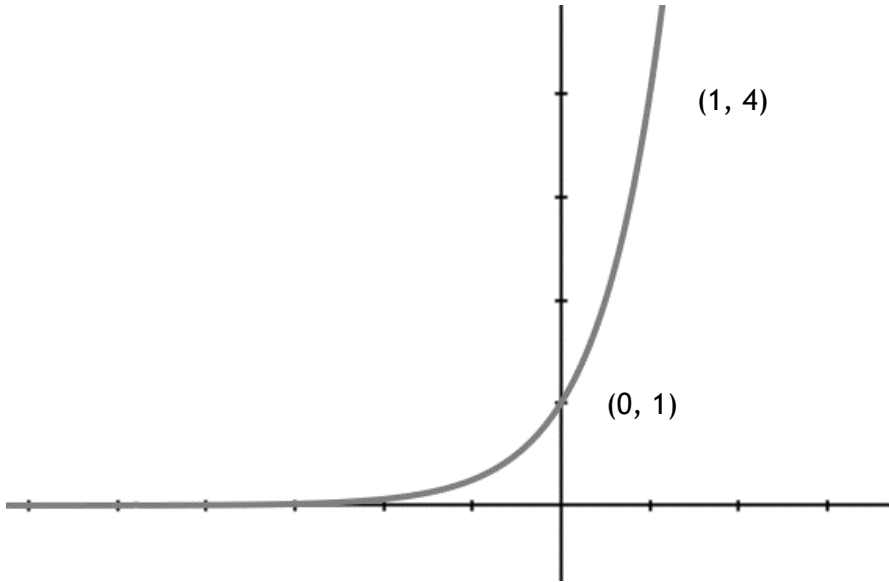
$$\text{So, } f^{-1}(x) = \frac{x - 5}{3}$$

b) $g^{-1}(7) = 2$

2016 Paper 1 Question 10, (2)

The point (1, 0) on the graph of $f(x)$ is transformed to (0, 1) on the graph of $f^{-1}(x)$

The point (4, 1) on the graph of $f(x)$ is transformed to (1, 4) on the graph of $f^{-1}(x)$



2017 Paper 1 Question 6, (3)

$$h(x) = x^3 + 7$$

$$\text{Let } y = h(x)$$

$$y = x^3 + 7$$

Swapping x & y gives

$$x = y^3 + 7$$

$$y^3 = x - 7$$

$$y = \sqrt[3]{x - 7}$$

$$\text{So, } h^{-1}(x) = \sqrt[3]{x - 7}$$

2018 Paper 1 Question 2, (3)

$$g(x) = \frac{1}{5}x - 4$$

$$y = \frac{1}{5}x - 4$$

$$5y = x - 20$$

$$x = 5y + 20$$

$$g^{-1}(x) = 5x + 20$$

2019 Paper 2 Question 8, (3) (1)

a) $f(x) = \sqrt[3]{x} + 8$

$$y = \sqrt[3]{x} + 8$$

$$y - 8 = \sqrt[3]{x}$$

$$x = (y - 8)^3$$

So, $f^{-1}(x) = (x - 8)^3$

b) The domain of $f^{-1}(x)$ is the range of $f(x)$

$$f(1) = 9$$

$$f(1000) = 18$$

So, the range of $f(x)$ is $9 \leq x \leq 18$

Hence, the domain of $f^{-1}(x)$ is $9 \leq x \leq 18, x \in \mathbb{R}$