Higher Mathematics

Exponential & Logarithm - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 20, (2)

The equation of the line is $\log_3 y = 2x$ since gradient $= 2$ and y-intercept $= 0$

$\log_3 y = 2x$

Rewriting as an exponential gives

$y = 3^{2x}$

2013 Paper 2 Question 5, (4)

$\log_5(3-2x) + \log_5(2+x) = 1$

$\log_5\left[(3-2x)(2+x)\right] = 1$

$5^1 = (3-2x)(2+x)$

$5 = 6 - x - 2x^2$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = -1, x = \dfrac{1}{2}$

2013 Paper 2 Question 9, (4) (3)

a) $p_t = p_0 e^{-kt}$

Since the concentration has halved $p_t = \dfrac{p_0}{2}$

$\dfrac{p_0}{2} = p_0 e^{-25k}$

$0.5 = e^{-25k}$

$\log_e 0.5 = \log_e e^{-25k}$

$\log_e 0.5 = -25k$

$k = \dfrac{\log_e 0.5}{-25}$

$k = 0.028$ to 2 significant figures

b) $p_t = p_0 e^{-kt}$

Let $t = 80$ and $k = 0.028$ Substitute to give

$p_t = p_0 e^{-0.028 \, x \, 80}$

$p_t = p_0 e^{-2.24}$

$p_t = 0.1065 \, p_0$

Rounding gives

$p_t = 0.11 \, p_0$

So, p_t is 11% of p_0

Therefore, the concentration has decreased by 89%

<u>2014 Paper 1 Question 3, (2)</u>

$\log_4 12 - \log_4 x = \log_4 6$

$\log_4 12 - \log_4 6 = \log_4 x$

$\log_4 \frac{12}{6} = \log_4 x$

$\log_4 2 = \log_4 x$

$x = 2$

<u>2014 Paper 1 Question 20, (2)</u>

$$2 - \log_5 \frac{1}{25} = 2 - (\log_5 1 - \log_5 25) = 2 - 0 + 2 = 4$$

<u>2014 Paper 1 Question 24, (5)</u>

$y = ka^x$

$\log_9 y = \log_9 (ka^x)$

$\log_9 y = \log_9 k + \log_9 a^x$

$\log_9 y = \log_9 k + x \log_9 a$

Rewrite in the format of a straight line

$\log_9 y = (\log_9 a)x + \log_9 k$

Consider the line shown in the graph

$$m = \frac{5-2}{6-0} = \frac{1}{2}$$

Y intercept $= 2$

So, by inspection we have

$\log_9 a = \frac{1}{2}$ $\qquad$ $\log_9 k = 2$

$a = 9^{\frac{1}{2}} = 3$ $\qquad$ $k = 9^2 = 81$

2015 Paper 1 Question 6, (3)

$$\log_6 12 + \frac{1}{3}\log_6 27 = \log_6 12 + \log_6 27^{\frac{1}{3}}$$

$$= \log_6 12 + \log_6 3$$

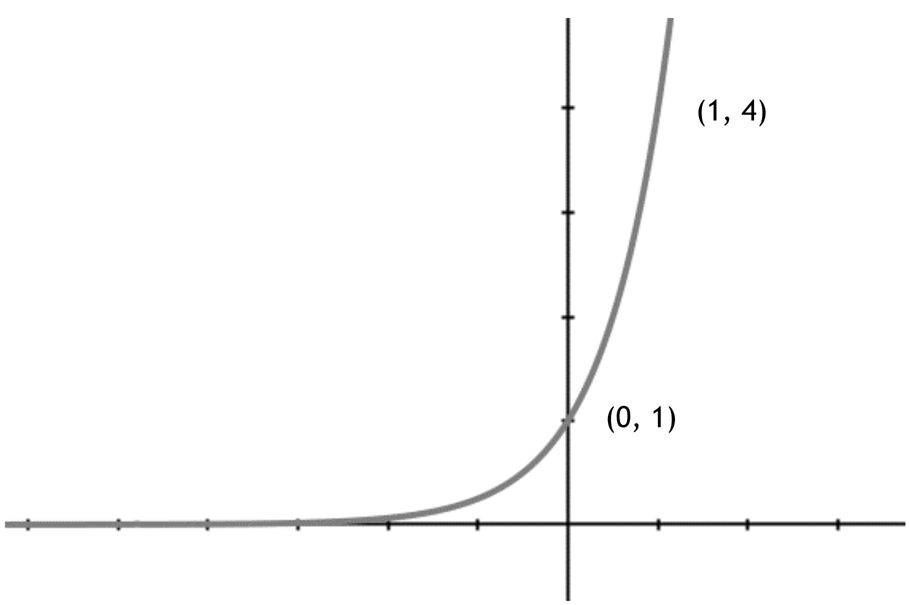
$$= \log_6 36$$

$$= 2$$

2016 Paper 1 Question 10, (2)

The point (1, 0) on the graph of $f(x)$ is transformed to (0, 1) on the graph of $f^{-1}(x)$

The point (4, 1) on the graph of $f(x)$ is transformed to (1, 4) on the graph of $f^{-1}(x)$



2016 Paper 1 Question 14, (1) (5)

a) $\log_5 25 = 2$ since $5^2 = 25$

b) $\log_4 x + \log_4(x-6) = \log_5 25$

$\log_4 x + \log_4(x-6) = 2$

$\log_4 x(x-6) = 2$

Rewriting as an exponential gives

$x(x-6) = 4^2$

$x^2 - 6x = 16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = -2, x = 8$

2016 Paper 2 Question 6, (1) (4)

a) $B(t) = 200e^{0.107t}$

At the start $t = 0$ giving $B(0) = 200 \bullet e^0 = 200$

b) When the number doubles there are 400 bacteria

Let $B(t) = 400$

$400 = 200e^{0.107t}$

$2 = e^{0.107t}$

$\log_e 2 = \log_e e^{0.107t}$

$\log_e 2 = 0.107t$

$t = \dfrac{\log_e 2}{0.107} = 6.48$ hours

2017 Paper 1 Question 12, (3)

$\log_a 36 - \log_a 4 = \dfrac{1}{2}$

$\log_a\left(\dfrac{36}{4}\right) = \dfrac{1}{2}$

$\log_a 9 = \frac{1}{2}$

Rewriting as an exponential to give

$a^{\frac{1}{2}} = 9$

$a = 9^2 = 81$

2017 Paper 2 Question 9, (5)

$y = kx^n$

$\log_2 y = \log_2 kx^n$

$\log_2 y = \log_2 k + \log_2 x^n$

$\log_2 y = \log_2 k + \text{n}\log_2 x$

Reorder to give

$\log_2 y = \text{n}\log_2 x + \log_2 k$

n is the line gradient $= \frac{3-0}{0-(-12)} = \frac{1}{4}$

$\log_2 k$ is the y intercept

$\log_2 k = 3$

$k = 2^3 = 8$

2018 Paper 1 Question 6, (3)

$\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{\frac{1}{3}}$

$= \log_5 250 - \log_5 2$

$= \log_5 \frac{250}{2}$

$= \log_5 125$

$= 3$

2018 Paper 1 Question 11, (2) (3)

a) $y = 1 - \log_3 x$

$y = -\log_3 x + 1$

The original graph needs to be reflected in the x-axis and moved up 1 unit

b) $1 - \log_3 x = \log_3 x$

$1 = 2\log_3 x$

$\log_3 x = \frac{1}{2}$

$3^{\frac{1}{2}} = x$

$x = \sqrt{x}$

2018 Paper 2 Question 11, (4) (2)

a) $P = 100(1 - e^{-kt})$

$50 = 100(1 - e^{-kt})$

$0.5 = 1 - e^{-kt}$

$e^{-kt} = 0.5$

$\log_e e^{3k} = \log_e 0.5$

$3k = \ln 0.5$

$k = \frac{\ln 0.5}{3}$

$k = -0.231$

b) $P = 100(1 - e^{-0.231t})$

$= 100(1 - e^{-0.231 \times 5})$

$= 100 - 100e^{-1.155}$

$= 100 - 31.51$

$= 68.49$

$= 68.5$

<u>2019 Paper 1 Question 14, (3) (3)</u>

a) $\log_{10}4 + 2\log_{10}5$

$= \log_{10}4 + \log_{10}5^2$

$= \log_{10}4 + \log_{10}25$

$= \log_{10}100$

$= 2$

b) $\log_2(7x - 2) - \log_2 3 = 5$

$$\log_2\left(\frac{7x - 2}{3}\right) = 5$$

$$2^5 = \frac{7x - 2}{3}$$

$$32 = \frac{7x - 2}{3}$$

$96 = 7x - 2$

$7x = 98$

$x = 14$

2019 Paper 2 Question 9, (1) (4)

a) Initially $t = 0$ to give

$$P_0 = 120e^{-0.0079\ x\ 0} = 120e^0 = 120$$

Initial electrical power is 120 Watts

b) 15% reduction $= 120\ x\ 0.85 = 102$

Substitute 102 to give

$$102 = 120e^{-0.0079t}$$

$$\frac{102}{120} = e^{-0.0079t}$$

$$0.85 = e^{-0.0079t}$$

$$\log_e 0.85 = \log_e e^{-0.0079t}$$

$$\ln 0.85 = -0.0079t$$

$$t = \frac{\ln 0.85}{-0.0079}$$

$$= 20.57$$

So, it would take 20.57 years

2019 Paper 2 Question 12, (5)

$y = ab^x$

Taking the log of both sides gives

$$\log_4 y = \log_4(ab^x)$$

$$= \log_4 a + \log_4 b^x$$

$$= \log_4 a + x\log_4 b$$

Reordering gives

$\log_4 y = \log_4 b(x) + \log_4 a$

This is in the format of a straight line where $\log_4 b$ represents the gradient and $\log_4 a$ represents the $y-$ intercept.

Using the points $(0, -1)$ $(3, 8)$ gives gradient $= \dfrac{8 - (-1)}{3 - 0} = 3$

So, $\log_4 b = 3$

$4^3 = b$

$b = 64$

From the graph the $y-$intercept is -1

So, $\log_4 a = -1$

$4^{-1} = a$

$a = \dfrac{1}{4}$