

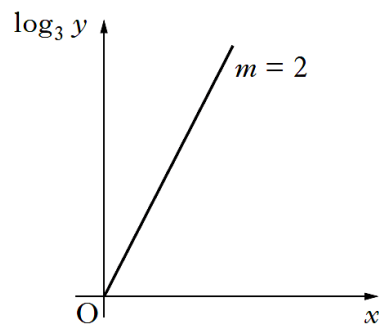
Higher Mathematics

Exponential & Logarithm - Questions - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 20, (2)

The graph of  $\log_3 y$  plotted against  $x$  is a line through the origin with gradient 2, as shown.



Express  $y$  in terms of  $x$ .

2013 Paper 2 Question 5, (4)

Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$

2013 Paper 2 Question 9, (4) (3)

The concentration of the pesticide,  $X_{pesto}$ , in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- $P_0$  is the initial concentration;
- $P_t$  is the concentration at time  $t$ ;
- $t$  is the time, in days, after the application of the pesticide.

(a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of  $X_{pesto}$  is 25 days, find the value of  $k$  to 2 significant figures.

(b) Eighty days after the initial application, what is the percentage decrease in concentration of  $X_{pesto}$ ?

2014 Paper 1 Question 3, (2)

If  $\log_4 12 - \log_4 x = \log_4 6$ , what is the value of  $x$ ?

2014 Paper 1 Question 20, (2)

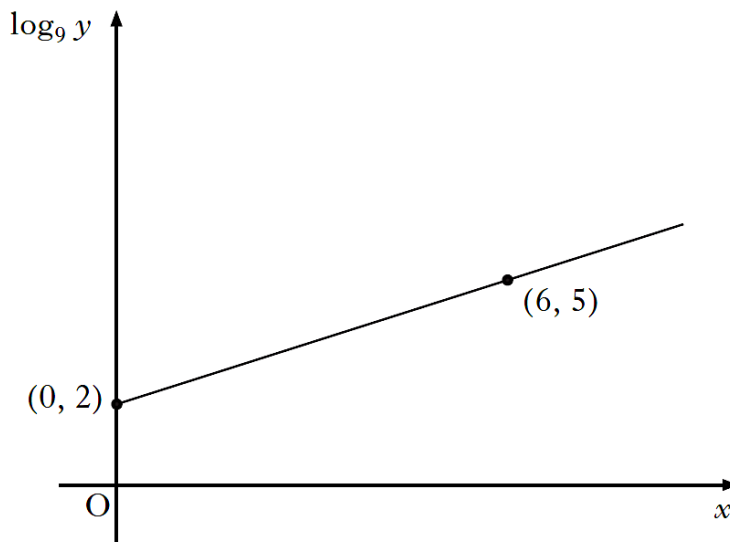
Evaluate  $2 - \log_5 \frac{1}{25}$ .

2014 Paper 1 Question 24, (5)

Two variables,  $x$  and  $y$ , are related by the equation

$$y = ka^x.$$

When  $\log_9 y$  is plotted against  $x$ , a straight line passing through the points  $(0, 2)$  and  $(6, 5)$  is obtained, as shown in the diagram.



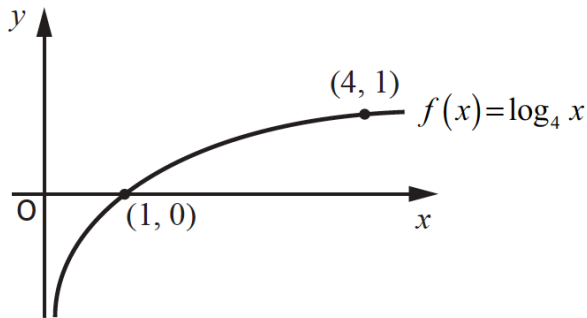
Find the values of  $k$  and  $a$ .

2015 Paper 1 Question 6, (3)

Evaluate  $\log_6 12 + \frac{1}{3} \log_6 27$ .

2016 Paper 1 Question 10, (2)

The diagram below shows the graph of the function  $f(x) = \log_4 x$ , where  $x > 0$ .



The inverse function,  $f^{-1}$ , exists.

On the diagram in your answer booklet, sketch the graph of the inverse function.

2016 Paper 1 Question 14, (1) (5)

- (a) Evaluate  $\log_5 25$ .
- (b) Hence solve  $\log_4 x + \log_4 (x - 6) = \log_5 25$ , where  $x > 6$ .

2016 Paper 2 Question 6, (1) (4)

Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0.107t},$$

where  $t$  represents the number of hours since the study began.

- (a) State the number of bacteria present at the start of the study.
- (b) Calculate the time taken for the number of bacteria to double.

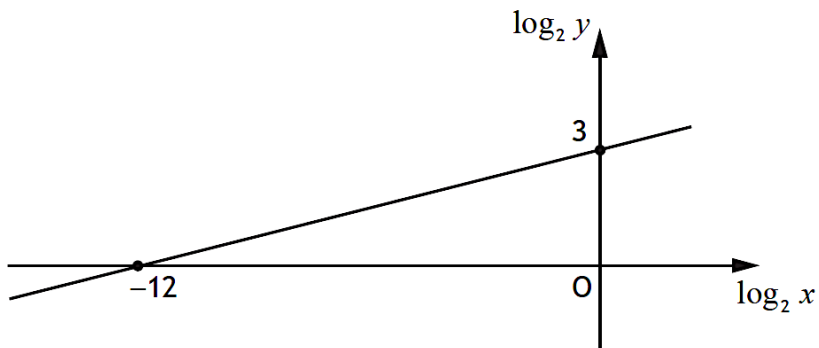
2017 Paper 1 Question 12, (3)

Given that  $\log_a 36 - \log_a 4 = \frac{1}{2}$ , find the value of  $a$ .

2017 Paper 2 Question 9, (5)

Two variables,  $x$  and  $y$ , are connected by the equation  $y = kx^n$ .

The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line as shown.



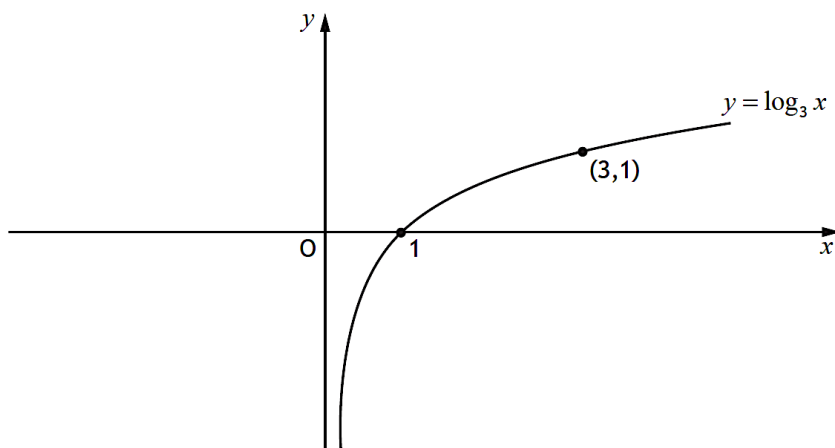
Find the values of  $k$  and  $n$ .

2018 Paper 1 Question 6, (3)

Find the value of  $\log_5 250 - \frac{1}{3} \log_5 8$ .

2018 Paper 1 Question 11, (2) (3)

The diagram shows the curve with equation  $y = \log_3 x$ .



- On the diagram in your answer booklet, sketch the curve with equation  $y = 1 - \log_3 x$ .
- Determine the exact value of the  $x$ -coordinate of the point of intersection of the two curves.

2018 Paper 2 Question 11, (4) (2)

A supermarket has been investigating how long customers have to wait at the checkout. During any half hour period, the percentage,  $P\%$ , of customers who wait for less than  $t$  minutes, can be modelled by

$$P = 100(1 - e^{-kt}), \text{ where } k \text{ is a constant.}$$

- (a) If 50% of customers wait for less than 3 minutes, determine the value of  $k$ .
- (b) Calculate the percentage of customers who wait for 5 minutes or longer.

2019 Paper 1 Question 14, (3) (3)

- (a) Evaluate  $\log_{10} 4 + 2\log_{10} 5$ .
- (b) Solve  $\log_2(7x - 2) - \log_2 3 = 5$ ,  $x \geq 1$ .

2019 Paper 2 Question 9, (1) (4)

Electricity on a spacecraft can be produced by a type of nuclear generator. The electrical power produced by this generator can be modelled by

$$P_t = 120e^{-0.0079t}$$

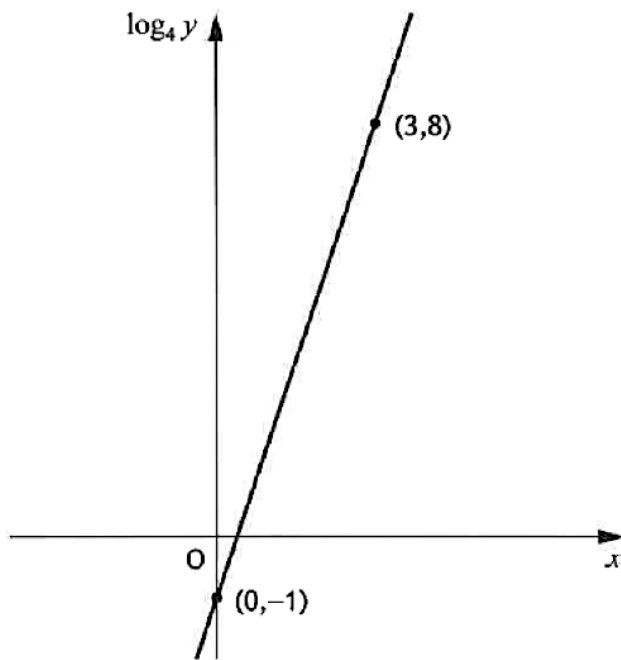
where  $P_t$  is the electrical power produced, in watts, after  $t$  years.

- (a) Determine the electrical power initially produced by the generator.
- (b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%.

2019 Paper 2 Question 12, (5)

Two variables,  $x$  and  $y$ , are connected by the equation  $y = ab^x$ .

The graph of  $\log_4 y$  against  $x$  is a straight line as shown.



Find the values of  $a$  and  $b$ .