Higher Mathematics

Domain - **Solutions** - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 13, (2)

Let $x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 3, x = 4$

So $x = 3, x = 4$ cannot be in the domain of $f(x)$

2014 Paper 1 Question 12, (2)

$$f(x) = \frac{6x}{x^2 + 6x - 16}$$

Restrictions on the domain of $f(x)$ occur where $x^2 + 6x - 16 = 0$ since we cannot divide by 0

Let $x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, x = 2$

So, the restrictions on the domain of f are $x = -8, x = 2$

2015 Paper 2 Question 2, (2) (3) (2)

a) $f\big(g(x)\big) = f((1+x)(3-x)+2)$

$= 10 + (1+x)(3-x) + 2$

Simplifying gives

$f\big(g(x)\big) = 15 + 2x - x^2$

b) $f\big(g(x)\big) = -x^2 + 2x + 15$

$= -\left(x^2 - 2x\right) + 15$

Completing the square of the inside of the bracket

$$= -\left[(x-1)^2 - 1\right] + 15$$

$$= -(x-1)^2 + 16$$

c) $h(x) = \dfrac{1}{f(g(x))} = \dfrac{1}{-(x-1)^2 + 16}$

Restrictions on the domain of $h(x)$ occur where $-(x-1)^2 + 16 = 0$

Let $-(x-1)^2 + 16 = 0$

$$(x-1)^2 = 16$$

$$x - 1 = 4 \qquad x - 1 = -4$$

$$x = 5 \qquad x = -3$$

So, $x = -3, x = 5$ cannot be in the domain of $h(x)$

<u>2019 Paper 1 Question 12, (2) (1)</u>

a) $f\left(g(x)\right) = f\left(5 - x\right) = \dfrac{1}{\sqrt{5 - x}}$

b) $f(g(x))$ is undefined where $5 - x \leq 0$ since you cannot square root a negative and you cannot divide by zero.

So, $5 - x \leq 0$

$$5 \leq x$$

$$x \geq 5$$

<u>2019 Paper 2 Question 8, (3) (1)</u>

a) $f(x) = \sqrt[3]{x} + 8$

$$y = \sqrt[3]{x} + 8$$

$y - 8 = \sqrt[3]{x}$

$x = (x - 8)^3$

So, $f^{-1}(x) = (x - 8)^3$

b) The domain of $f^{-1}(x)$ is the range of $f(x)$

$f(1) = 9$

$f(1000) = 18$

So, the range of $f(x)$ is $9 \leq x \leq 18$

Hence, the domain of $f^{-1}(x)$ is $9 \leq x \leq 18, x \in \mathbb{R}$