

Higher Mathematics

Differentiation - Solutions - 2013-2019

Marks are indicated in brackets after each question number

2013 Paper 1 Question 2, (2)

$$y = x^2 - 4x + 7$$

$$\frac{dy}{dx} = 2x - 4$$

$$\text{Gradient of tangent at } (5, 12) = 2 \cdot 5 - 4 = 6$$

Using $y - b = m(x - a)$ gives

$$y - 12 = 6(x - 5)$$

$$y - 12 = 6x - 30$$

$$y = 6x - 28$$

2013 Paper 1 Question 18, (2)

$$y = \sin(x^2 - 3)$$

$$\frac{dy}{dx} = \cos(x^2 - 3) \cdot 2x$$

$$= 2x \cos(x^2 - 3)$$

2014 Paper 1 Question 8, (2)

$$\text{Let } f(x) = (4 - 9x^4)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}} \cdot (-36x^3)$$

$$= -18x^3(4 - 9x^4)^{\frac{1}{2}}$$

2014 Paper 1 Question 9, (2)

Note there are various ways to solve this problem. This approach uses the derivative to find max values

$$\text{Let } f(x) = 5\sin 2x + 5\sqrt{3}\cos 2x$$

$$f'(x) = 10\cos 2x - 10\sin 2x$$

For maximum values $f'(x) = 0$

$$10\cos 2x - 10\sin 2x = 0$$

$$\cos 2x - \sin 2x = 0$$

$$\cos 2x = \sin 2x$$

$$1 = \frac{\sin 2x}{\cos 2x} = \tan 2x$$

$$\text{From exact values } 2x = \frac{\pi}{6}$$

So, the maximum value occurs where $2x = \frac{\pi}{6}$

Substituting $2x = \frac{\pi}{6}$ gives

$$f(x) = 5\sin\frac{\pi}{6} + 5\sqrt{3}\cos\frac{\pi}{6} = \frac{5}{2} + \frac{15}{2} = 10$$

2014 Paper 1 Question 21, (6) (2)

a) $y = 3x^2 - x^3$

$$\frac{dy}{dx} = 6x - 3x^2$$

For stationary points $\frac{dy}{dx} = 0$

$$6x - 3x^2 = 0$$






$$3x(2 - x) = 0$$

$$x = 0, x = 2$$

When $x = 0, y = 0$

When $x = 2, y = 4$

So, stationary points are (0, 0) (2, 4)

x	-1	0	1	2	5
$f'(x)$	+	0	-	0	+
<i>Shape</i>					

So $(0, 0)$ is a minimum turning point and $(2, 4)$ is a maximum turning point

b) For the y intercept $x = 0$ giving

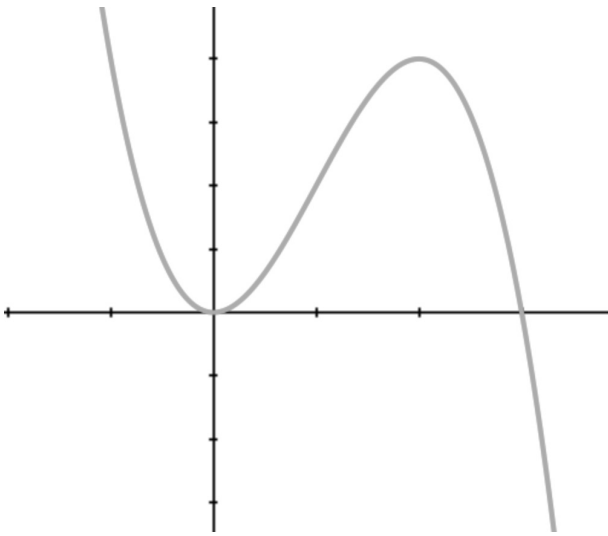
$$y = 3 \cdot 0^2 - 0^3 = 0$$

For the x intercept $y = 0$ giving

$$0 = 3x^2 - x^3$$

$$0 = x^2(3 - x)$$

$$x = 0, x = 3$$



2014 Paper 2 Question 2, (4)

$$y = x^4 - 2x^3 + 5$$

$$\frac{dy}{dx} = 4x^3 - 6x^2$$

$$\text{When } x = 2, \frac{dy}{dx} = 32 - 24 = 8$$

So, the gradient of the tangent at $x = 2$ is 8

$$\text{When } x = 2, y = 2^4 - 2 \cdot 2^3 + 5 = 5$$

Using $y - b = m(x - a)$ with $(2, 5)$ gives

$$y - 5 = 8(x - 2)$$

Rearranging gives

$$y = 8x - 11$$

2015 Paper 1 Question 2, (4)

$$y = 2x^3 + 3$$

$$\frac{dy}{dx} = 6x^2$$

$$\text{When } x = -2, \frac{dy}{dx} = 6 \cdot (-2)^2 = 24$$

So, gradient of tangent at $x = -2$ is 24

$$\text{When } x = -2, y = 2 \cdot (-2)^3 + 3 = -13$$

So, $(-2, -13)$ lies on the curve

Using $y - b = m(x - a)$ we have

$$y - (-13) = -2(x - (-13))$$

$$y + 13 = -2x - 26$$

$$y = -2x - 39$$

2015 Paper 1 Question 7, (4)

$$f(x) = \sqrt{x} \left(3x - \frac{2}{x\sqrt{x}} \right)$$

$$= x^{\frac{1}{2}} \left(3x - \frac{2}{x \cdot x^{\frac{1}{2}}} \right)$$

$$= x^{\frac{1}{2}} \left(3x - \frac{2}{x^{\frac{3}{2}}} \right)$$

$$= x^{\frac{1}{2}} (3x - 2x^{-\frac{3}{2}})$$

$$= 3x^{\frac{3}{2}} - 2x^{-1}$$

$$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 2x^{-2} = \frac{9}{2}x^{\frac{1}{2}} + \frac{2}{x^2}$$

$$f'(4) = \left(\frac{9}{2} \cdot 2 \right) + \frac{2}{16} = 9 + \frac{1}{8} = \frac{73}{8}$$

2016 Paper 1 Question 2, (3)

$$y = 12x^3 + 8\sqrt{x}$$

$$y = 12x^3 + 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$$

2016 Paper 1 Question 9, (4) (2)

a) $f(x) = x^3 + 3x^2 - 24x$

$$f'(x) = 3x^2 + 6x - 24$$

For stationary points $f'(x) = 0$

Let $f'(x) = 0$






$$3x^2 + 6x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

$$x = -4, x = 2$$

b)

x	-10	-4	1	2	10
$f'(x)$	+	0	-	0	+
<i>Shape</i>					

So, f is strictly increasing for $x < -4$ and for $x > 2$

So, $-4 > x > 2$

2016 Paper 2 Question 10, (2) (1)

$$\text{a) } y = (x^2 + 7)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 7)^{-\frac{1}{2}} \cdot 2x$$

$$= x(x^2 + 7)^{-\frac{1}{2}}$$

$$= \frac{x}{(x^2 + 7)^{\frac{1}{2}}}$$

$$= \frac{x}{\sqrt{x^2 + 7}}$$

$$\text{b) } \int \frac{4x}{\sqrt{x^2 + 7}} dx$$

$$= 4 \int \frac{x}{\sqrt{x^2 + 7}} dx$$

$$= 4(x^2 + 7)^{\frac{1}{2}} \text{ using the result from a)}$$

2017 Paper 1 Question 3, (2)

$$y = (4x - 1)^{12}$$

$$\frac{dy}{dx} = 12(4x - 1)^{11} \cdot 4$$

$$= 48(4x - 1)^{11}$$

2017 Paper 1 Question 8, (3)

$$d(t) = \frac{1}{2t} = \frac{t^{-1}}{2} = \frac{1}{2}t^{-1}$$

$$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$$

$$d'(5) = -\frac{1}{2 \cdot 5^2} = -\frac{1}{50}$$

So, the rate of change of $d(t)$ when $t = 5$ is $-\frac{1}{50}$

2017 Paper 2 Question 7, (4) (3)

a) $y = 6x - 2\sqrt{x^3}$

$$y = 6x - 2x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$$

For stationary points $\frac{dy}{dx} = 0$

$$6 - 3x^{\frac{1}{2}} = 0$$

$$6 = 3x^{\frac{1}{2}}$$

$$2 = x^{\frac{1}{2}}$$

Squaring both sides gives

$$4 = x$$

b) When $x = 4$, $y = 8$

When $x = 1$, $y = 4$

When $x = 9$, $y = 0$

So, the maximum value of y on $1 \leq x \leq 9$ is 8

And the minimum value is 0

2018 Paper 1 Question 3, (3)

$$h(x) = 3\cos(2x)$$

$$h'(x) = -6\sin(2x)$$

$$h'\left(\frac{\pi}{6}\right) = -6\sin\left(2x \frac{\pi}{6}\right)$$

$$= -6\sin\left(\frac{\pi}{3}\right)$$

$$= -6x \frac{\sqrt{3}}{2}$$

$$= -3\sqrt{3}$$

2018 Paper 1 Question 7, (1) (3) (4)

a) (0, 5)

b) $y = x^3 - 3x^2 + 2x + 5$

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

$$m_{tan} = 3(0)^2 - 6(0) + 2$$

$$= 2$$

$$y = 2x + 5$$

c) $2x + 5 = x^3 - 3x^2 + 2x + 5$

$$x^3 - 3x^2 = 0$$

$$x^2(x - 3) = 0$$

$$x = 0, x = 3$$

When $x = 3$, $y = (2 \times 3) + 5$

$$= 11$$

$$Q = (3, 11)$$

2018 Paper 1 Question 15, (5)

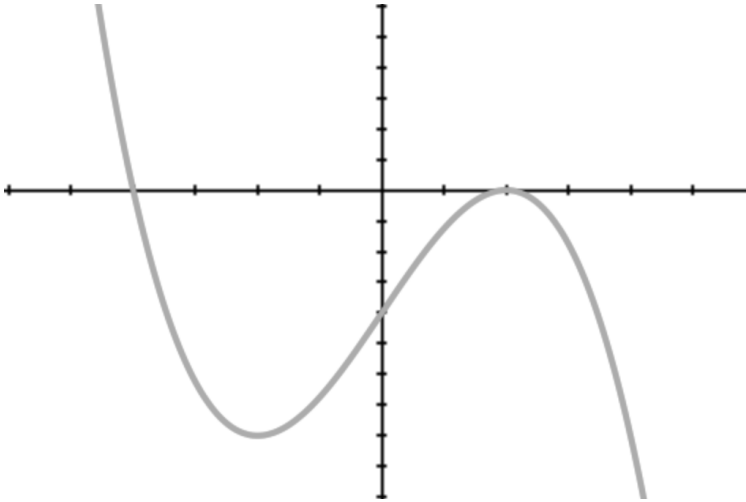
Since $(x + 4)$ is a factor, $x = -4$ is a root

Since $x = 2$ is a repeated root, $(x - 2)$ is a repeated factor

So, the equation of the function can be written $f(x) = k(x + 4)(x - 2)(x - 2)$

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$

Considering all of the above information and assuming a k value (which cannot be determined from the information) a possible graph is:



2018 Paper 2 Question 3, (3)

$$f(x) = x^3 - 7x - 6$$

$$f'(x) = 3x^2 - 7$$

$$f'(2) = 5$$

Since $f'(2) > 0$, the function is increasing

2018 Paper 2 Question 9, (6)

$$P(x) = 2x + \frac{128}{x}$$

$$= 2x + 128x^{-1}$$

$$P'(x) = 2 - 128x^{-2} = 2 - \frac{128}{x^2}$$

For minimum values, $P'(x) = 0$




$$2 - \frac{128}{x^2} = 0$$

$$2x^2 - 128 = 0$$

$$x^2 = 64$$

$$x = -8, x = 8$$

Discard $x = -8$ since x cannot be negative

x	1	8	10
$P'(x)$	-	0	+
<i>Shape</i>			

Substitute $x = 8$ to give

$$P(x) = (2 \times 8) + \frac{128}{8}$$

$$= 16 + 16$$

$$= 32$$

2019 Paper 1 Question 1, (4)

$$y = \frac{1}{2}x^4 - 2x^3 + 6$$

$$\frac{dy}{dx} = 2x^3 - 6x^2$$

For stationary points $\frac{dy}{dx} = 0$ giving

$$2x^3 - 6x^2 = 0$$

$$2x^2(x - 3) = 0$$

$$2x^2 = 0$$

$$x = 0$$

$$x - 3 = 0$$

$$x = 3$$

2019 Paper 1 Question 6, (3)

$$y = \frac{1}{(1 - 3x)^5}$$

$$= (1 - 3x)^{-5}$$

$$\frac{dy}{dx} = -5(1 - 3x)^{-6} \cdot (-3)$$

$$= 15(1 - 3x)^{-6}$$