

Higher Mathematics

Composite Functions - Solutions - 2013-2019

2013 Paper 1 Question 1, (2)

$$\begin{aligned}g(f(x)) &= g(x^2 + 1) = 3(x^2 + 1) - 4 \\ &= 3x^2 + 3 - 4 \\ &= 3x^2 - 1\end{aligned}$$

2014 Paper 2 Question 3, (2)

$$\begin{aligned}\text{a) } f(g(x)) &= f(x + 3) = (x + 3)(x + 3 - 1) + q \\ &= (x + 3)(x + 2) + q \\ &= x^2 + 5x + 6 + q\end{aligned}$$

$$\text{b) } f(g(x)) = x^2 + 5x + 6 + q = x^2 + 5x + (6 + q)$$

$$a = 1, b = 5, c = 6 + q$$

For equal roots $b^2 - 4ac = 0$

$$25 - 4 \cdot 1 \cdot (6 + q) = 0$$

$$25 - 24 - 4q = 0$$

$$q = \frac{1}{4}$$

2015 Paper 1 Question 5, (2) (1)

$$\text{a) } g(x) = 6 - 2x$$

$$y = 6 - 2x$$

Interchange x & y to give

$$x = 6 - 2y$$

Solve for y

$$y = 3 - \frac{1}{2}x$$

$$\text{So, } g^{-1}(x) = 3 - \frac{1}{2}x$$

b) $g(g^{-1}(x)) = x$ since g and g^{-1} are inverses of each other

2015 Paper 2 Question 2, (2) (3) (2)

$$\begin{aligned} \text{a) } f(g(x)) &= f((1+x)(3-x) + 2) \\ &= 10 + (1+x)(3-x) + 2 \end{aligned}$$

Simplifying gives

$$f(g(x)) = 15 + 2x - x^2$$

$$\begin{aligned} \text{b) } f(g(x)) &= -x^2 + 2x + 15 \\ &= -(x^2 - 2x) + 15 \end{aligned}$$

Completing the square of the inside of the bracket

$$\begin{aligned} &= - \left[(x-1)^2 - 1 \right] + 15 \\ &= -(x-1)^2 + 16 \end{aligned}$$

$$\text{c) } h(x) = \frac{1}{f(g(x))} = \frac{1}{-(x-1)^2 + 16}$$

Restrictions on the domain of $h(x)$ occur where $-(x-1)^2 + 16 = 0$

$$\text{Let } -(x-1)^2 + 16 = 0$$

$$(x-1)^2 = 16$$

$$x-1 = 4 \quad x-1 = -4$$

$$x = 5 \quad x = -3$$

So, $x = -3$, $x = 5$ cannot be in the domain of $h(x)$

2016 Paper 1 Question 12, (2) (3)

$$\text{a) } h(x) = f(g(x)) = f(3-x) = 2(3-x)^2 - 4(3-x) + 5 = 2x^2 - 8x + 11$$

$$\begin{aligned} \text{b) } h(x) &= 2x^2 - 8x + 11 = 2(x^2 - 4x) + 11 \\ &= 2 \left[(x-2)^2 - 4 \right] + 11 \end{aligned}$$

$$= 2(x - 2)^2 - 8 + 11$$

$$= 2(x - 2)^2 + 3$$

2017 Paper 1 Question 1, (1) (2)

a) $f(g(x)) = f(2\cos x) = 5 \cdot 2 \cos x = 10\cos x$

$$f(g(0)) = 10 \cdot \cos 0 = 10$$

b) $g(f(x)) = g(5x) = 2\cos(5x)$

2018 Paper 2 Question 6, (2) (1)

a) i) $f(g(x)) = 3 + \cos(2x)$

ii) $g(f(x)) = 2(3 + \cos x)$

$$= 6 + 2\cos x$$

2019 Paper 1 Question 12, (2) (1)

a) $f(g(x)) = f(5 - x) = \frac{1}{\sqrt{5 - x}}$

b) $f(g(x))$ is undefined where $5 - x \leq 0$ since you cannot square root a negative and you cannot divide by zero.

So, $5 - x \leq 0$

$$5 \leq x$$

$$x \geq 5$$