2019 National 5 Paper 1

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Marks are indicated in brackets after each question

Question 1, (2)

$f(x) = 5x^3$

$f(-2) = 5 \bullet (-2)^3$

$= 5 \bullet -8$

$= -40$

Question 2, (2)

$\frac{3}{8} \, x \, 1\frac{5}{7} = \frac{3}{8} \, x \, \frac{12}{7}$

$= \frac{36}{56}$

$= \frac{9}{14}$

Question 3, (3)

$(x + 5)(2x^2 - 7x - 3)$

$= 2x^3 - 7x^2 - 3x + 10x^2 - 35x - 15$

$= 2x^3 + 3x^2 - 38x - 15$

Question 4, (3)

Arc Length $= \frac{angle}{360} \, x \, \pi \, x \, d$

$= \frac{240}{360} \, x \, 3.14 \, x \, 60$

$= \frac{2}{3} \, x \, 3.14 \, x \, 60$

$= 40 \, x \, 3.14$

$$= (40\ x\ 3) + (40\ x\ 0.1) + (40\ x\ 0.04)$$
$$= 120 + 4 + 1.6$$
$$= 125.6\ cm$$

Question 5, (3) (2)

a) Reordering gives

3 3 4 4 5 6 7 9 10

Median $= 5$

$Q_3 = 8$

$Q_1 = 3.5$

Semi-IQR $= \dfrac{8\ -\ 3.5}{2} = \dfrac{4.5}{2} = 2.25$

b) The median temperature was higher in Endoch than Grantford meaning that on average Endoch had higher midday temperatures over the nine day period.

The semi-interquartile range was lower in Endoch than Grantford meaning that the temperatures recorded over the nine day period were more consistent there.

Question 6, (3) (1)

a) Choose two points that lie on the line of best fit, $(1.5,\ 14),\ (3.5,\ 8)$

Gradient $= \dfrac{14\ -\ 8}{1.5\ -\ 3.5}$

$$= \frac{6}{-2}$$
$$= -3$$

Using $y - b = m(x - a)$ with $(1.5,\ 14)$ gives

$y - 14 = -3(x - 1.5)$

$y - 14 = -3x + 4.5$

$y = -3x + 18.5$

$F = -3E + 18.5$

b) Substituting $E = 1.1$ gives

$$F = (-3\ x\ 1.1) + 18.5$$
$$= -3.3 + 18.5$$
$$= 15.2$$

15.2 Kilometres per litre

Question 7, (3)

$$A = \frac{1}{2}h(x + y)$$
$$2A = h(x + y)$$
$$2A = hx + hy$$
$$2A - hy = hx$$
$$x = \frac{2A - hy}{h}$$

Question 8, (1) (1) (4)

a) $7c + 3g = 215$

b) $5c + 4g = 200$

c) $7c + 3g = 215$ (1)

$5c + 4g = 200$ (2)

Multiply (1) by 4 and multiply (2) by 3 to give

$28c + 12g = 860$ (3)

$15c + 12g = 600$ (4)

(3) - (4) gives

$13c = 260$

$c = 20$

Question 9, (1) (1) (1)

a) $x = 4$

b) i) $a = -4$

ii) $b = 20$

Question 10, (1) (2)

a) $\overrightarrow{PQ} = \overrightarrow{PR} + \overrightarrow{RQ}$

$$= \begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

b) $\overrightarrow{MQ} = \overrightarrow{MP} + \overrightarrow{PQ}$

$$= \frac{1}{2}\overrightarrow{RP} + \overrightarrow{PQ}$$

$$= -\frac{1}{2}\overrightarrow{PR} + \overrightarrow{PQ}$$

$$= -\frac{1}{2}\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

Question 11, (3)

All of the angles at $O = 360 \div 5 = 72°$

$AOB = 72°$

$FOB = 180 - 72 = 108°$

$$OFB = \frac{180 - 108}{2}$$

$$= 36°$$

Question 12, (3)

$$\frac{\sqrt{2}}{\sqrt{40}} = \frac{\sqrt{2}}{\sqrt{2}\,\sqrt{20}} = \frac{1}{\sqrt{20}} = \frac{1}{\sqrt{20}}\,x\,\frac{\sqrt{20}}{\sqrt{20}} = \frac{\sqrt{20}}{20} = \frac{\sqrt{4}\,\sqrt{5}}{20} = \frac{2\sqrt{5}}{20} = \frac{\sqrt{5}}{10}$$

Question 13, (2)

$x-$ co-ordinate of $A = 180 - 45 = 135$

$y-$ co-ordinate of $A = -1\,x\,3 = -3$

Co-ordinates of $A = (135,\ -3)$

Question 14, (3)

$$\frac{x}{2} - 1 = \frac{3-x}{5}$$

Multiply through the equation by 10 to give

$$\frac{10x}{2} - 10 = \frac{30-10x}{5}$$

$$5x - 10 = 6 - 2x$$

$$7x = 16$$

$$x = \frac{16}{7}$$

Question 15, (1) (4)

a) $h = 12t - 5t^2$

Substitute $t = 2$ to give

$h = (12\,x\,2) - 5(2^2)$

$= 24 - 20$

$= 4$

4 metres

b) Substitute $h = -17$ to give

$-17 = 12t - 5t^2$

$5t^2 - 12t - 17 = 0$

$(5t - 17)(t + 1) = 0$

$5t - 17 = 0$

$$t = \frac{17}{5}$$

$$t = 3.4$$

$$t + 1 = 0$$

$$t = -1$$

Since t represents *time* this solution can be discarded

The ball will hit the sea after 3.4 seconds

2019 National 5 Paper 2

Question 1, (3)

$80{,}000 \; x \; 1.15 = 92{,}000$

$92{,}000$ blankets

Question 2, (2)

$$\left| \underline{p} \right| = \sqrt{6^2 + 27^2 + (-18)^2}$$

$$= \sqrt{1089}$$

$$= 33$$

Question 3, (2)

$$\text{Area} = \frac{1}{2} \; x \; 45 \; x \; 70 \; x \text{Sin} 129°$$

$$= 1{,}224 \; cm^2$$

Question 4, (2)

$$\left(3.6 \; x \; 10^{-6}\right) \; x \; 0.08$$

$$= 2.9 \; x \; 10^{-7}$$

Question 5, (2)

$A = (3, 0, 0)$

$B = (3, 3, 8)$

Question 6, (3)

$3x^2 + 9x - 2 = 0$

$a = 3, b = 9, c = -2$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(3)(-2)}}{2(3)} = \frac{-9 \pm \sqrt{81 + 24}}{6}$$

$$= \frac{-9 \pm \sqrt{105}}{6}$$

$$x = \frac{-9 + \sqrt{105}}{6} = 0.2 \text{ and } x = \frac{-9 - \sqrt{105}}{6} = -3.2$$

Question 7, (3)

The smallest angle is at vertex Z

Using the Cosine Rule gives

$$Cos\, Z = \frac{8.5^2 + 7.2^2 - 6.3^2}{2\,(8.5)\,(7.2)}$$

$$= \frac{84.4}{122.4}$$

$$Z = cos^{-1}\left(\frac{84.4}{122.4}\right)$$

$$= 46.4°$$

Question 8, (5)

Volume of hemi-sphere $= \frac{1}{2}\, x\, \frac{4}{3}\pi r^3$

$$= \frac{1}{2}\, x\, \frac{4}{3}\, x\, \pi\, x\, (12^3)$$

$$= 3,619.11\ cm^3$$

Volume of cylinder $= \pi r^2 h$

$$= \pi\, x\, (12^2)\, x\, (70 - 12)$$

$$= 26,238.58\ cm^3$$

Volume of bollard $= 3,619.11 + 26,238.58 = 29,857.69\ cm^3$

Question 9, (3)

$977.85 = 102.5\%$

$1\% = 977.85 \div 102.5 = 9.54$

$100\% = 9.54\ x\ 100 = 954$

So, $£954$ is the price if she had paid on time

$£977.85 - £954 = £23.85$

She could have saved $£23.85$

Question 10, (2)

$x^2 + 10x - 15 = \left(x + 5\right)^2 - 40$

Question 11, (4)

The length of B to C is given by $1500 - 600 - 650 = 250\ m$

$650^2 = 422{,}500$

$600^2 + 250^2 = 422{,}500$

Since $600^2 + 250^2 = 650^2$ a triangle with short sides 600 & 250 and long side 650 is a right-angled triangle by the Converse of Pythagoras' Theorem.

So, ABC is a right-angled triangle, meaning that B is due east of A since C is due north of B.

Question 12, (3) (3)

a) Linear Scale Factor $= \frac{30}{50}$

Area Scale Factor $= (\frac{30}{50})^{2} = 0.36$

Area $= 2{,}750\ x\ 0.36 = 990\ cm^2$

b) Area $= \frac{angle}{360}\ x\ \pi r^2$

Let the angle $ACB = x$

$2{,}750 = \frac{x}{360} x\ \pi\ x\ 50^2$

$2{,}750 = \frac{2{,}500\pi x}{360}$

$x = \frac{2{,}750\ x\ 360}{2{,}500\pi}$

$x = 126.1°$

Question 13, (3)

Gradient $= \frac{9\ -\ 4p^2}{6\ -\ 4p}$

$= \frac{(3\ -\ 2p)(3\ +\ 2p)}{2(3\ -\ 2p)}$

$= \frac{3\ +\ 2p}{2}$

Question 14, (3)

$5\cos x + 2 = 1$

$5\cos x = -\ 1$

$\cos x = -\frac{1}{5}$

$\cos x = -\ 0.2$

$cos^{-1}(0.2) = 78°$

From CAST solutions lie in quadrants 2 & 3, giving

$x = 180 - 78 = 102°$

$x = 180 + 78 = 258°$

Question 15, (3)

$$\frac{4}{x-2} - \frac{3}{x+5} = \frac{4(x+5)}{(x-2)(x+5)} - \frac{3(x-2)}{(x-2)(x+5)}$$

$$= \frac{4(x+5) - 3(x-2)}{(x-2)(x+5)}$$

$$= \frac{4x + 20 - 3x + 6}{(x-2)(x+5)}$$

$$= \frac{x + 26}{(x-2)(x+5)}$$

Question 16, (3)

$$\frac{a^4 \, x \, 3a}{\sqrt{a}} = \frac{3a^5}{a^{\frac{1}{2}}} = 3a^{5-\frac{1}{2}} = 3a^{\frac{9}{2}}$$

Question 17, (2)

$$(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$$

$$= sin^2 x + 2\sin x \cos x + cos^2 x$$

$$= sin^2 x + cos^2 x + 2\sin x \cos x$$

$[sin^2 x + cos^2 x = 1$ from Trig Identities]

$$= 2\sin x \cos x + 1$$

Question 18, (4)

Create a right angled triangle TSB

Since TS & SB are the radius of the circle they have length $7.5 \, cm$

By Pythagoras, $TB = \sqrt{7.5^2 + 7.5^2}$

$$= 10.6 \, cm$$

TB is the radius of the larger circle, so TD also has length 10.6 cm

So, height $= 10.6 + 15 = 25.6 \, cm$

Question 19, (5)

Angle $B = 180 - 52 - 34 = 94°$

Using the Sine Rule gives

$$\frac{b}{\sin B} = \frac{k}{\sin K} = \frac{m}{\sin M}$$

$$\frac{350}{\sin 94°} = \frac{k}{\sin 52°} = \frac{m}{\sin 34°}$$

$$m = \frac{350 \sin 34°}{\sin 94°} = 196 \, metres$$

Add a vertical line from B to the ground at point G to make a right-angled triangle BKG with angles $52°,\ 38°,\ 90°$.

Using the Sine Rule gives

$$\frac{b}{\sin B} = \frac{k}{\sin K} = \frac{g}{\sin G}$$

$$\frac{b}{\sin 38°} = \frac{k}{\sin 52°} = \frac{196}{\sin 90°}$$

$$k = \frac{196 \sin 52°}{\sin 90°} = 154$$

So, the height is 154 *metres* above the ground