2019 Higher Paper 1

Click to jump to question:
Paper 1: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Paper 2: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Question 1, (4)

$y = \frac{1}{2}x^4 - 2x^3 + 6$

$\frac{dy}{dx} = 2x^3 - 6x^2$

For stationary points $\frac{dy}{dx} = 0$ giving

$2x^3 - 6x^2 = 0$

$2x^2(x-3) = 0$

$2x^2 = 0$

$x = 0$

$x - 3 = 0$

$x = 3$

Question 2, (3)

$x^2 + (k-5)x + 1 = 0$

For equal roots $b^2 - 4ac = 0$

$a = 1, b = k - 5, c = 1$

$b^2 - 4ac = (k-5)^2 - 4(1)(1)$

$= k^2 - 10k + 25 - 4$

$= k^2 - 10k + 21$

$k^2 - 10k + 21 = 0$

$(k-3)(k-7) = 0$

$k = 3, k = 7$

Question 3, (2)

Radius of $C_1 = \sqrt{(-3)^2 + 1^2 + 26}$

$= \sqrt{36}$

$= 6$

Radius of C_2 is the same as the radius of $C_1 = 6$

So, the equation of C_2 is $(x-4)^2 + (y+2)^2 = 6^2$

Question 4, (3) (1)

a) Let $u_0 = 6, u_1 = 9, u_2 = 11$

$u_1 = mu_0 + c$

$9 = 6m + c$

$c = 9 - 6m \qquad (1)$

$u_2 = mu_1 + c$

$11 = 9m + c \qquad (2)$

Substitute (1) into (2) giving

$11 = 9m + 9 - 6m$

$2 = 3m$

$m = \frac{2}{3}$

From 1) $c = 9 - 6\left(\frac{2}{3}\right) = 5$

b) $u_3 = \frac{2}{3}(11) + 5$

$= \frac{22}{3} + \frac{15}{3}$

$= \frac{37}{3}$

$$u_4 = \frac{2}{3}\left(\frac{37}{3}\right) + 5$$

$$= \frac{74}{9} + 5$$

$$= \frac{74}{9} + \frac{45}{9}$$

$$= \frac{119}{9}$$

Question 5, (3) (1)

a) $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix} = 4\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$4\overrightarrow{AB} = 3\overrightarrow{BC}$ so $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel but since B is a common point, A, B, & C are collinear.

b) 3 : 4

Question 6, (3)

$$y = \frac{1}{(1-3x)^5}$$

$$= (1-3x)^{-5}$$

$$\frac{dy}{dx} = -5(1-3x)^{-6} \bullet (-3)$$

$$= 15(1-3x)^{-6}$$

Question 7, (4)

$\tan 30° = m = \dfrac{1}{\sqrt{3}}$

Using $y - b = m(x - a)$ gives

$$y + 4 = \frac{1}{\sqrt{3}}(x - 0)$$

$$y + 4 = \frac{1}{\sqrt{3}}x$$

$$y = \frac{1}{\sqrt{3}}x - 4$$

Question 8, (1) (3)

a) $\displaystyle\int_{-1}^{2} \left[\left(x^2 + 2x + 3\right) - (2x^2 + x + 1)\right] dx$

$$= \int_{-1}^{2} \left(-x^2 + x + 2\right) dx$$

b) $\displaystyle\int_{-1}^{2} \left(-x^2 + x + 2\right) dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$

$$= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$= \dfrac{27}{6}$ after simplification

So, the area is $\dfrac{27}{6}\, units^2$

Question 9, (1) (3) (2)

a) i) $\underline{u} \bullet \underline{v} = p(2p + 16) + (-2 \bullet -3) + (4 \bullet 6)$

$= 2p^2 + 16p + 30$

ii) For perpendicular vectors $\underline{u} \bullet \underline{v} = 0$, giving

$2p^2 + 16p + 30 = 0$

$p^2 + 8p + 15 = 0$

$(p + 5)(p + 3) = 0$

$p = -5, p = 3$

b) If $\underline{u}$ and $\underline{v}$ are parallel, there exists a number k such that $\underline{u} = k\underline{v}$ i.e. one vector is a multiple of the other.

Comparing the y component of $\underline{u}$ and $\underline{v}$ we have $-2 = k(-3)$, giving $k = \frac{2}{3}$

So, $p = \frac{2}{3}(2p + 16)$

$3p = 4p + 32$

$p = -32$

Question 10, (1) (1)

a) $a = 3$

b) $y = kf(x) + a$

Using the points $(2, -1)$ and $(2, 5)$ for substitution gives

$5 = k(-1) + 3$

$2 = -k$

$k = -2$

Question 11, (4)

$$\int_0^{9} \cos\left(3x - \frac{\pi}{6}\right) dx = \left[\frac{1}{3}\sin(3x - \frac{\pi}{6})\right]_0^{\frac{\pi}{9}}$$

$$= \frac{1}{3}\sin\left(3\left(\frac{\pi}{9}\right) - \frac{\pi}{6}\right) - \frac{1}{3}\sin(0 - \frac{\pi}{6})$$

$$= \frac{1}{3}\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) - \frac{1}{3}\sin(- \frac{\pi}{6})$$

$$= \frac{1}{6} - (- \frac{1}{6})$$

$$= \frac{1}{3}$$

Question 12, (2) (1)

a) $f\left(g(x)\right) = f\left(5 - x\right) = \dfrac{1}{\sqrt{5 - x}}$

b) $f(g(x))$ is undefined where $5 - x \leq 0$ since you cannot square root a negative and you cannot divide by zero.

So, $5 - x \leq 0$

$5 \leq x$

$x \geq 5$

Question 13, (1) (1) (3)

a) i) $AC = \sqrt{(\sqrt{5})^2 - 1^2} = 2$

$\cos p = \dfrac{2}{\sqrt{5}}$

ii) $AD = 2 + 1 = 3$

$\cos q = \dfrac{3}{\sqrt{10}}$

b) $\sin(p+q) = \sin p\cos q + \cos p\sin q$

$$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) + (\frac{2}{\sqrt{5}})(\frac{1}{\sqrt{10}})$$

$$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{5}{5\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

Question 14, (3) (3)

a) $\log_{10}4 + 2\log_{10}5$

$= \log_{10}4 + \log_{10}5^2$

$= \log_{10}4 + \log_{10}25$

$= \log_{10}100$

$= 2$

b) $\log_2(7x-2) - \log_2 3 = 5$

$$\log_2\left(\frac{7x - 2}{3}\right) = 5$$

$$2^5 = \frac{7x - 2}{3}$$

$$32 = \frac{7x - 2}{3}$$

$96 = 7x - 2$

$7x = 98$

$x = 14$

Question 15, (4) (1)

a) $\sin 2x + 6\cos x = 0$

$2\sin x\cos x + 6\cos x = 0$

$2\cos x\,(\sin x + 3) = 0$

$2\cos x = 0$

$\cos x = 0$

$x = 90°,\ 270°$

$\sin x + 3 = 0$

$\sin x = -3$

No solutions since $-1 \leq \sin x \leq 1$

b) $\sin 4x + 6\cos 2x = 0$

Compare with the equation in a) to see that the angles are doubled meaning that the graph of this function is compressed by a factor of 2. This means that the solutions to b) are half those of a) giving $x = 45°,\ 135°$

Also, additional solutions are introduced because of the period change of the function in b). These are found by adding $360°$ to the solutions from a) and then dividing by 2.

$x = \dfrac{90 + 360}{2} = 225°$ and $x = \dfrac{270 + 360}{2} = 315°$

Question 16, (2) (4)

a) $P = (4, k)$ $C = (1, -2)$

Using the distance formula gives, distance $= \sqrt{(4-1)^2 + (k+2)^2}$

$= \sqrt{9 + k^2 + 4k + 4}$

$= \sqrt{k^2 + 4k + 13}$

b) From the equation the circle has radius $= \sqrt{25} = 5$

If P lies outside the circle then the distance between C & P is greater than 5

So, $\sqrt{k^2 + 4k + 13} > 5$

$k^2 + 4k + 13 > 25$

$k^2 + 4k - 12 > 0$

$(k+6)(k-2) > 0$

By considering the roots and shape of the graph of $y = k^2 + 4k - 12$ we have

$k < -6$ and $k > 2$

Question 17 (3) (2)

a) $(\sin x - \cos x)^2 = sin^2x + cos^2x - 2\sin x\cos x$

$= 1 - 2\sin x\cos x$ since $sin^2x + cos^2x = 1$

$= 1 - \sin 2x$

b) $\int (\sin x - \cos x)^2 dx = \int (1 - \sin 2x)\, dx$ using part a)

$= x + \frac{1}{2}\cos 2x + c$

2019 Higher Paper 2

Question 1, (3) (3) (2)

a) Since D is the mid-point of AC it has co-ordinates $(-4, -3)$

$m_{BD} = \frac{-3+8}{-4-11} = -\frac{1}{3}$

Using $y - b = m(x - a)$ with $(11, -8)$ gives

$y + 8 = -\frac{1}{3}(x - 11)$

$3y + 24 = -x + 11$

$3y = -x - 13$ (1)

b) $m_{BC} = \dfrac{6\ +\ 8}{-3\ -\ 11} = \dfrac{14}{-14} = -1$

$m_{AE} = 1$ since AE & BC are perpendicular $\rightarrow m_{BC} \bullet m_{AE} = -1$

Using $y - b = m(x - a)$ with $\left(-5,\ -12\right)$ gives

$y + 12 = x + 5$

$y = x - 7$ (2)

c) By substituting (2) into (1) we have

$3(x - 7) = -x - 13$

$3x - 21 = -x - 13$

$4x = 8$

$x = 2$

When $x = 2, y = 2 - 7 = -5$ using equation (2)

So, the point of intersection is $\left(2,\ -5\right)$

Question 2, (4)

$$\int (6\sqrt{x} - 4x^{-3} + 5)\, dx$$

$$= \int \left(6x^{\frac{1}{2}} - 4x^{-3} + 5\right) dx$$

$$= \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{-2}}{-2} + 5x + c$$

$$= 4x^{\frac{3}{2}} + 2x^{-2} + 5x + c$$

Question 3, (1) (2)

a) $\overrightarrow{BE} = -\underline{p} + \underline{r}$

b) $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AB} + \overrightarrow{BF}$

$= -\underline{r} + \underline{p} + \frac{3}{4}\underline{q}$

Question 4, (1) (1) (2)

a) $U_{n+1} = aU_n + b$

$U_{n+1} = 0.973U_n + 30$

$a = 0.973 \qquad b = 30$

b) i) Since $-1 < 0.973 < 1$ the recurrence relation generates a sequence which has a limit, so the mouse population will tend towards that limit over time

ii) Limit $= \dfrac{30}{1 - 0.973} = 1{,}111.111$

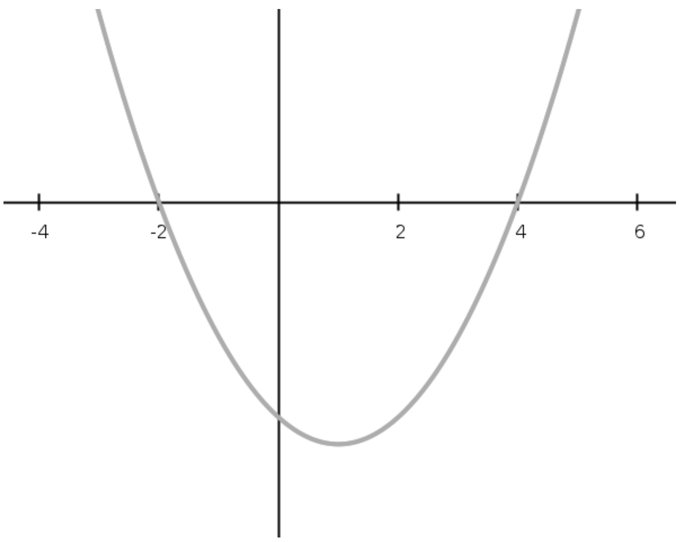
$= 1{,}100$ to the nearest hundred

So, the long term population will be 1,100 to the nearest hundred

Question 5, (2)

The stationary points of the graph of $y = g(x)$ are the roots of the graph of $y = g'(x)$.

So, the roots of $y = g'(x)$ are at $(-2, 0)$ $(4, 0)$ giving a sketch as follows:



Question 6, (4) (3)

a) $2\cos x - 3\sin x = k\cos(x + a)$

$= k\cos x\cos a - k\sin x\sin a$

$= k\cos a\cos x - k\sin a\sin x$

$2 = k\cos a \qquad 3 = k\sin a$

$k = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\frac{3}{2} = \tan a$

$a = tan^{-1}\left(\frac{3}{2}\right) = 56.3°$ Check that this is in the correct quadrant using CAST

So, $2\cos x - 3\sin x = \sqrt{13}\cos(x + 56.3)°$

b) $2\cos x - 3\sin x = 3$

Using the result from a) gives

$\sqrt{13}\cos(x + 56.3)° = 3$

$\cos\left(x + 56.3\right) = \frac{3}{\sqrt{13}}$

$cos^{-1}\left(\frac{3}{\sqrt{13}}\right) = 33.69°$

Using CAST, solutions to $\cos\left(x + 56.3\right) = \frac{3}{\sqrt{13}}$ should lie in quadrants 1 & 4

$x + 56.3 = 33.69$

$x = 33.69 - 56.3 = -22.61$ which is not in the range $0 \leq x \leq 360°$

But since Cosine repeats values every $360°$ we have a solution given by

$x = -22.61 + 360 = 337.39°$

$x + 56.3 = 360 - 33.69$

$x = 270°$

Question 7, (3) (3)

a) $-6x^2 + 24x - 25 = -6\left(x^2 - 4x\right) - 25$

$= -6\left[(x-2)^2 - 4\right] - 25$

$= -6(x-2)^2 + 24 - 25$

$= -6(x-2)^2 - 1$

b) $f(x) = -2x^3 + 12x^2 - 25x + 9$

$f'(x) = -6x^2 + 24x - 25$

Using the result from a) gives

$f'(x) = -6(x-2)^2 - 1$

Since $(x-2)^2 > 0$ for all x, $-6(x-2)^2 < 0$ for all x

So, $f'(x) < 0$ for all x meaning that $f(x)$ is strictly decreasing for all x

Question 8, (3) (1)

a) $f(x) = \sqrt[3]{x} + 8$

$y = \sqrt[3]{x} + 8$

$y - 8 = \sqrt[3]{x}$

$x = (x-8)^3$

So, $f^{-1}(x) = (x-8)^3$

b) The domain of $f^{-1}(x)$ is the range of $f(x)$

$f(1) = 9$

$f(1000) = 18$

So, the range of $f(x)$ is $9 \leq x \leq 18$

Hence, the domain of $f^{-1}(x)$ is $9 \leq x \leq 18, x \in \mathbb{R}$

Question 9, (1) (4)

a) Initially $t = 0$ to give

$P_0 = 120e^{-0.0079 \, x \, 0} = 120e^0 = 120$

Initial electrical power is 120 Watts

b) 15% reduction $= 120 \, x \, 0.85 = 102$

Substitute 102 to give

$102 = 120e^{-0.0079t}$

$\dfrac{102}{120} = e^{-0.0079t}$

$0.85 = e^{-0.0079t}$

$\log_e 0.85 = \log_e e^{-0.0079t}$

$\ln 0.85 = -0.0079t$

$t = \dfrac{\ln 0.85}{-0.0079}$

$= 20.57$

So, it would take 20.57 years

Question 10, (2) (5)

a)

- 3	3	10	1	- 8	- 6
		- 9	- 3	6	6
	3	1	- 2	- 2	0

Since the remainder is 0, $(x + 3)$ is a factor

b) $3x^4 + 10x^3 + x^2 - 8x - 6$

$= (x + 3)(3x^3 + x^2 - 2x - 2)$

We need to factorise $3x^3 + x^2 - 2x - 2$. Let $g(x) = 3x^3 + x^2 - 2x - 2$

By inspection, $(x - 1)$ is a factor since $g(1) = 0$

1	3	1	- 2	-2
		3	4	2
	3	4	2	0

So, $3x^4 + 10x^3 + x^2 - 8x - 6 = (x+3)(x-1)(3x^2 + 4x + 2)$

Note this does not factorise any further

Question 11, (3) (6)

a) Sides $= 4 \bullet 3x \bullet h$

$= 12xh$

Top $= (3x)^2 - x^2$

$= 9x^2 - x^2$

$= 8x^2$

Base $= 8x^2$

Inside of Tunnel $= 4xh$

Total $= 16x^2 + 16xh$

Volume of cuboid including central tunnel $= l \bullet b \bullet h$

$= (3x) \bullet (3x) \bullet h$

$= 9x^2h$

Volume of tunnel $= l \bullet b \bullet h$

$= x^2h$

Volume of cuboid without central tunnel $= 9x^2h - x^2h$

$= 8x^2h$

Since volume $= 2000$ we have

$$2000 = 8x^2h$$

$$h = \frac{2000}{8x^2}$$

Substituting into (1) gives Total Surface Area $= 16x^2 + 16x(\frac{2000}{8x^2})$

$$= 16x^2 + \frac{4000}{x}$$

So, $A(x) = 16x^2 + \frac{4000}{x}$

b) $A(x) = 16x^2 + \frac{4000}{x}$

$$= 16x^2 + 4000x^{-1}$$

For minimum areas $A'(x) = 0$ giving

$$32x - 4000x^{-2} = 0$$

$$32x - \frac{4000}{x^2} = 0$$

$$32x^3 - 4000 = 0$$

$$x^3 = \frac{4000}{32}$$

$$x = \sqrt[3]{\frac{4000}{32}}$$

$$x = 5$$

x	1	5	10
$A'(x)$	−	0	+
Shape	\	—	/

The nature table confirms that $x = 5$ gives a minimum value of $A(x)$

$$A(5) = 16 \bullet 5^2 + \frac{4000}{5}$$
$$= 400 + 800$$
$$= 1200\ cm^2$$

Question 12, (5)

$y = ab^x$

Taking the log of both sides gives

$$\log_4 y = \log_4(ab^x)$$
$$= \log_4 a + \log_4 b^x$$
$$= \log_4 a + x\log_4 b$$

Reordering gives

$$\log_4 y = \log_4 b(x) + \log_4 a$$

This is in the format of a straight line where $\log_4 b$ represents the gradient and $\log_4 a$ represents the $y-$intercept.

Using the points $(0,\ -1)\ \ (3, 8)$ gives gradient $= \dfrac{8\ -\ (-1)}{3\ -\ 0} = 3$

So, $\log_4 b = 3$

$$4^3 = b$$
$$b = 64$$

From the graph the $y-$ intercept is -1

So, $\log_4 a = -1$

$$4^{-1} = a$$
$$a = \frac{1}{4}$$

Question 13, (5)

$$f'(x) = 3x^2 - 16x + 11$$
$$\int \left(3x^2 - 16x + 11\right)\, dx = x^3 - 8x^2 + 11x + c$$

So, $f(x) = x^3 - 8x^2 + 11x + c$

Substitute $(7, 0)$ to give

$0 = 7^3 - 8 \bullet 7^2 + 11 \bullet 7 + c$

Simplifying gives $c = -28$

So, $f(x) = x^3 - 8x^2 + 11x - 28$

Question 14, (4)

Let θ be the angle between $\underline{u}$ $\underline{v}$

$$\underline{u} \bullet \left(\underline{u} + \underline{v}\right) = \underline{u} \bullet \underline{u} + \underline{u} \bullet \underline{v}$$

$$= \left|\underline{u}\right| \left|\underline{u}\right| \cos 0 + \left|\underline{v}\right| \left|\underline{v}\right| \cos\theta$$

$$= (4\ x\ 4\ x\ 1) + (4\ x\ 5\ x \cos\theta)$$

$$= 16 + 20\cos\theta$$

Since $\underline{u} \bullet \left(\underline{u} + \underline{v}\right) = 21$ we have

$$16 + 20\cos\theta = 21$$

$$20\cos\theta = 5$$

$$\cos\theta = \frac{5}{20}$$

$$\theta = cos^{-1}(\frac{5}{20})$$

$$\theta = 75.5°$$

Question 15, (3) (1) (3)

a) $m_{cp} = \dfrac{13 - 12}{5 - 8} = -\dfrac{1}{3}$

So, $m_{tan} = 3$

Using $y - b = m(x - a)$ with $(5, 13)$ gives

$y - 13 = 3(x - 5)$

$y - 13 = 3x - 15$

$y = 3x - 2$

b) $T = (0, -2)$

c) Midpoint of $PT = (\frac{5}{2}, \frac{15}{2})$

Distance $PT = \sqrt{(5-0)^2 + (13+2)^2}$

$= \sqrt{25 + 225}$

$= \sqrt{250}$

So, the circle radius is $\frac{\sqrt{250}}{2}$

Circle equation is $(x - \frac{5}{2})^2 + (y - \frac{15}{2})^2 = (\frac{\sqrt{250}}{2})^2$

Simplifying gives $(x - \frac{5}{2})^2 + (y - \frac{15}{2})^2 = 62.5$