

2018 Higher Paper 1

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Question 1, (3)

Mid-point of PQ = (1, 2)

Let $s = (1, 2)$

$$m_{RS} = \frac{6 - 2}{3 - 1} = 2$$

Using $y - b = m(x - a)$ with (1, 2) gives

$$y - 2 = 2(x - 1)$$

$$y - 2 = 2x - 2$$

$$y = 2x$$

Question 2, (3)

$$g(x) = \frac{1}{5}x - 4$$

$$y = \frac{1}{5}x - 4$$

$$5y = x - 20$$

$$x = 5y + 20$$

$$g^{-1}(x) = 5x + 20$$

Question 3, (3)

$$h(x) = 3\cos(2x)$$

$$h'(x) = -6\sin(2x)$$

$$h'\left(\frac{\pi}{6}\right) = -6\sin\left(2 \times \frac{\pi}{6}\right)$$

$$= -6 \sin\left(\frac{\pi}{3}\right)$$

$$= -6 \times \frac{\sqrt{3}}{2}$$

$$= -3\sqrt{3}$$

Question 4, (4)

$$x^2 + y^2 - 12x - 6y - 23 = 0$$

Circle centre = (6, 3)

Let $C = (6, 3)$

$$m_{KC} = \frac{3 - (-5)}{6 - 8} = \frac{8}{-2} = -4$$

$$m_{perp} = \frac{1}{4}$$

Using $y - b = m(x - a)$ with (6, 3) gives

$$y - 3 = \frac{1}{4}(x - 6)$$

$$4y - 12 = x - 6$$

$$4y = x + 6$$

$$y = \frac{1}{4}x + \frac{6}{4}$$

Question 5, (1) (1)

$$\text{a) } \frac{8}{10} = \frac{4}{5}$$

$$\text{b) } \frac{4}{5} \text{ of } (9 - 4)$$

$$= \frac{4}{5} \times 5$$

$$= 4$$

$$t = 4$$

Question 6, (3)

$$\begin{aligned}\log_5 250 - \frac{1}{3} \log_5 8 \\ &= \log_5 250 - \log_5 8^{\frac{1}{3}} \\ &= \log_5 250 - \log_5 2 \\ &= \log_5 \frac{250}{2} \\ &= \log_5 125 \\ &= 3\end{aligned}$$

Question 7, (1) (3) (4)

a) (0, 5)

$$\text{b) } y = x^3 - 3x^2 + 2x + 5$$

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

$$\begin{aligned}m_{tan} &= 3(0)^2 - 6(0) + 2 \\ &= 2\end{aligned}$$

$$y = 2x + 5$$

$$\text{c) } 2x + 5 = x^3 - 3x^2 + 2x + 5$$

$$x^3 - 3x^2 = 0$$

$$x^2(x - 3) = 0$$

$$x = 0, x = 3$$

$$\begin{aligned}\text{When } x = 3, y &= (2 \times 3) + 5 \\ &= 11\end{aligned}$$

$$Q = (3, 11)$$

Question 8, (2)

$$y - \sqrt{3}x + 5 = 0$$

$$y = \sqrt{3}x - 5$$

$$\text{Gradient} = \sqrt{3}$$

$$\text{So, } \tan\theta = \sqrt{3}$$

$$\theta = 60^\circ \text{ from exact values}$$

Question 9, (1) (2)

$$\text{a) } \vec{BC} = -\underline{t} + \underline{u} = \underline{u} - \underline{t}$$

$$\begin{aligned} \text{b) } \vec{MD} &= \vec{MC} + \vec{CA} + \vec{AD} \\ &= \frac{1}{2}\vec{BC} + \vec{CA} + \vec{AD} \\ &= \frac{1}{2}(\underline{u} - \underline{t}) - \underline{u} + \underline{v} \\ &= \frac{1}{2}\underline{u} - \frac{1}{2}\underline{v} - \underline{u} + \underline{v} \\ &= \underline{v} - \frac{1}{2}\underline{u} - \frac{1}{2}\underline{t} \end{aligned}$$

Question 10, (4)

$$\frac{dy}{dx} = 6x^2 - 3x + 4$$

$$\int 6x^2 - 3x + 4 \, dx$$

$$= 2x^3 - \frac{3}{2}x^2 + 4x + c$$

$$\text{So, } y = 2x^3 - \frac{3}{2}x^2 + 4x + c$$

Substitute $x = 2$, $y = 14$ to give

$$14 = 2(2^3) - \frac{3}{2}(2^2) + 4(2) + c$$

Rearrange to give

$$c = -4$$

$$\text{So, } y = 2x^3 - \frac{3}{2}x^2 + 4x - 4$$

Question 11, (2) (3)

$$\text{a) } y = 1 - \log_3 x$$

$$y = -\log_3 x + 1$$

The original graph needs to be reflected in the x-axis and moved up 1 unit

$$\text{b) } 1 - \log_3 x = \log_3 x$$

$$1 = 2\log_3 x$$

$$\log_3 x = \frac{1}{2}$$

$$3^{\frac{1}{2}} = x$$

$$x = \sqrt{x}$$

Question 12, (1) (3)

$$\text{a) } \underline{a} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$2\underline{a} + \underline{b} = 2 \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$$

$$\text{b) } |2\underline{a} + \underline{b}| = \sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$$

$$36 + 9 + 16 + 8p + p^2 = 49$$

$$p^2 + 8p + 12 = 0$$

$$(p + 6)(p + 2) = 0$$

$$p = -6, p = -2$$

Question 13, (3) (1) (3)

$$\text{a) i) } \sin(2x) = 2\sin x \cos x$$

$$= 2x \frac{2}{\sqrt{11}} x \frac{\sqrt{7}}{\sqrt{11}}$$

$$= \frac{4\sqrt{7}}{11}$$

$$\text{ii) } \cos(2x) = 1 - 2\sin^2 x$$

$$= 1 - 2\left(\frac{2}{\sqrt{11}}\right)^2$$

$$= 1 - 2\left(\frac{4}{11}\right)$$

$$= 1 - \frac{8}{11}$$

$$= \frac{3}{11}$$

$$\begin{aligned}
\text{b) } \sin(3x) &= \sin(2x + x) \\
&= \sin(2x)\cos x + \cos(2x)\sin x \\
&= \left(\frac{4\sqrt{7}}{11} x \frac{\sqrt{7}}{11} \right) + \left(\frac{3}{11} x \frac{2}{11} \right) \\
&= \frac{28}{11\sqrt{11}} + \frac{6}{11\sqrt{11}} \\
&= \frac{34}{11\sqrt{11}}
\end{aligned}$$

Question 14, (5)

$$\begin{aligned}
\int_{-4}^9 \frac{1}{\sqrt[3]{(2x+9)^2}} dx &= \int_{-4}^9 (2x+9)^{-\frac{2}{3}} dx \\
&= \left[\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3} \times 2} \right]_{-4}^9 \\
&= \left[\frac{3}{2} (2x+9)^{\frac{1}{3}} \right]_{-4}^9 \\
&= \left(\frac{3}{2} \left(27^{\frac{1}{3}} \right) \right) - \left(\frac{3}{2} \left(1^{\frac{1}{3}} \right) \right) \\
&= \frac{9}{2} - \frac{3}{2} \\
&= \frac{6}{2} \\
&= 3
\end{aligned}$$

Question 15, (5)

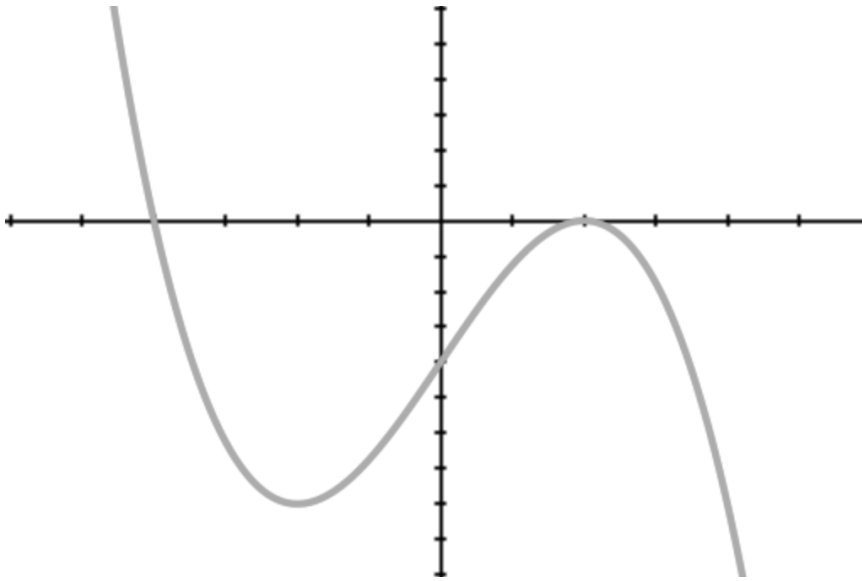
Since $(x + 4)$ is a factor, $x = -4$ is a root

Since $x = 2$ is a repeated root, $(x - 2)$ is a repeated factor

So, the equation of the function can be written $f(x) = k(x + 4)(x - 2)(x - 2)$

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$

Considering all of the above information and assuming a k value (which cannot be determined from the information) a possible graph is:



2018 Higher Paper 2

Question 1, (4)

$$\begin{aligned} & \int_{-1}^3 3 + 2x - x^2 \, dx \\ &= \left[3x + x^2 - \frac{1}{3}x^3 \right]_{-1}^3 \\ &= (9 + 9 - 9) - \left(-3 + 1 + \frac{1}{3} \right) \\ &= 11 - \frac{1}{3} \\ &= \frac{33}{3} - \frac{1}{3} \\ &= \frac{32}{3} \end{aligned}$$

$$\text{Area} = 10.67 \text{ units}^2$$

Question 2, (1) (4)

$$\text{a) } \underline{u} \cdot \underline{v} = 7 + 32 - 15 = 24$$

$$\text{b) } |\underline{u}| = \sqrt{1 + 16 + 9} = \sqrt{26}$$

$$|\underline{v}| = \sqrt{49 + 64 + 25} = \sqrt{138}$$

$$\cos\theta = \frac{24}{\sqrt{26} \sqrt{138}}$$

$$\theta = \cos^{-1}\left(\frac{24}{\sqrt{26} \sqrt{138}}\right)$$

$$= 66.38^\circ$$

Question 3, (3)

$$f(x) = x^3 - 7x - 6$$

$$f'(x) = 3x^2 - 7$$

$$f'(2) = 5$$

Since $f'(2) > 0$, the function is increasing

Question 4, (3)

$$\begin{aligned} -3x^2 - 6x + 7 &= -3(x^2 + 2x) + 7 \\ &= -3\left[(x+1)^2 - 1\right] + 7 \\ &= -3(x+1)^2 + 3 + 7 \\ &= -3(x+1)^2 + 10 \end{aligned}$$

Question 5, (3) (2) (2)

a) Mid-point $PQ = (6, 1)$

$$m_{PQ} = \frac{4 + 2}{3 - 9} = \frac{6}{-6} = -1$$

So, $m_{L1} = 1$

$$y - b = m(x - a)$$

$$y - 1 = x - 6$$

$$y = x - 5$$

b) $3y + x = 25$

$$3y = 25 - x$$

$$y = x - 5$$

$$3y = 3x - 15$$

Equating gives

$$3x - 15 = 25 - x$$

$$4x = 40$$

$$x = 10$$

When $x = 10$, $y = 10 - 5 = 5$

So, $C = (10, 5)$

c) $C = (10, 5)$

$$P = (3, 4)$$

$$\begin{aligned} \text{Distance} &= \sqrt{(10 - 3)^2 + (5 - 4)^2} \\ &= \sqrt{50} \end{aligned}$$

$$(x - 10)^2 + (y - 5)^2 = 50$$

Question 6, (2) (1) (6)

a) i) $f(g(x)) = 3 + \cos(2x)$

ii) $g(f(x)) = 2(3 + \cos x)$
 $= 6 + 2\cos x$

b) $3 + \cos(2x) = 6 + 2\cos x$

$$\cos(2x) - 2\cos x - 3 = 0$$

$$2\cos^2 x - 1 - 2\cos x - 3 = 0$$

$$2\cos^2 x - 2\cos x - 4 = 0$$

$$\cos^2 x - \cos x - 2 = 0$$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x - 2 = 0$$

$$\cos x = 2$$

No solutions

$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$x = \pi$$

Question 7, (2) (2) (1) (3) (1)

a) i)

2	2	-3	-3	2
		4	2	-2
	2	1	-1	0

Zero remainder shows that $(x - 2)$ is a factor

ii) $2x^3 - 3x^2 - 3x + 2 = (x - 2)(2x^2 + x - 1)$
 $= (x - 2)(2x - 1)(x + 1)$

$$\text{b) } u_5 = 2a - 3$$

$$\begin{aligned} u_6 &= au_5 - 1 \\ &= a(2a - 3) - 1 \\ &= 2a^2 - 3a - 1 \end{aligned}$$

$$\begin{aligned} u_7 &= au_6 - 1 \\ &= a(2a^2 - 3a - 1) - 1 \\ &= 2a^3 - 3a^2 - a - 1 \end{aligned}$$

$$\text{c) i) } 2a - 3 = 2a^3 - 3a^2 - a - 1$$

$$2a^3 - 3a^2 - 3a + 2 = 0$$

From a) ii) we have

$$(a - 2)(2a - 1)(a + 1) = 0$$

$$a = -1, \frac{1}{2}, 2$$

But since there is a limit, $-1 < a < 1$

$$\text{So, } a = \frac{1}{2}$$

$$\text{ii) Limit} = \frac{b}{1 - a} = \frac{-1}{1 - \frac{1}{2}} = -2$$

Question 8, (4) (1) (2)

$$\begin{aligned} \text{a) } 2\cos x - \sin x &= k\cos(x - a) \\ &= k \cos x \cos a + k \sin x \sin a \\ &= k \cos a \cos x + k \sin a \sin x \end{aligned}$$

$$2 = k \cos a$$

$$-1 = k \sin a$$

$$k = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

$$\tan a = -\frac{1}{2}$$

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.4^\circ$$

From CAST $a = 360 - 26.4 = 333.6^\circ$

$$2\cos x - \sin x = \sqrt{5}\cos(x - 333.6)^\circ$$

$$\begin{aligned} \text{b) i) } 6\cos x - 3\sin x &= 3(2\cos x - \sin x) \\ &= 3\sqrt{5}\cos(x - 333.6)^\circ \end{aligned}$$

Considering the graph of this function gives a minimum value of $-3\sqrt{5}$

$$\begin{aligned} \text{ii) } 3\sqrt{5}\cos(x - 333.6)^\circ &= -3\sqrt{5} \\ \cos(x - 333.6)^\circ &= -1 \\ x - 333.6 &= 180 \\ x &= 180 + 333.6 = 513.4 \\ x &= 513.4 - 360 = 153.4^\circ \end{aligned}$$

Question 9, (6)

$$\begin{aligned} P(x) &= 2x + \frac{128}{x} \\ &= 2x + 128x^{-1} \end{aligned}$$

$$P'(x) = 2 - 128x^{-2} = 2 - \frac{128}{x^2}$$

For minimum values, $P'(x) = 0$




$$2 - \frac{128}{x^2} = 0$$

$$2x^2 - 128 = 0$$

$$x^2 = 64$$

$$x = -8, x = 8$$

Discard $x = -8$ since x cannot be negative

x	1	8	10
$P'(x)$	-	0	+
<i>Shape</i>			

Substitute $x = 8$ to give

$$\begin{aligned}P(x) &= (2 \times 8) + \frac{128}{8} \\ &= 16 + 16 \\ &= 32\end{aligned}$$

So, the minimum value of $P(x)$ is 32

Question 10, (4)

$$x^2 + (m - 3)x + m = 0$$

$$a = 1, b = m - 3, c = m$$

$$\begin{aligned}b^2 - 4ac &= (m - 3)^2 - 4(1)(m) \\ &= m^2 - 6m + 9 - 4m \\ &= m^2 - 10m + 9\end{aligned}$$

Since there are two roots, $b^2 - 4ac > 0$

$$m^2 - 10m + 9 > 0$$

$$(m - 9)(m - 1) > 0$$

Consider the roots of $m^2 - 10m + 9$ which are at $m = 1, m = 9$

Consider the graph of $m^2 - 10m + 9$ giving

$$m < 1, m > 9$$

Question 11, (4) (2)

a) $P = 100(1 - e^{-kt})$

$$50 = 100(1 - e^{-kt})$$

$$0.5 = 1 - e^{-kt}$$

$$e^{-kt} = 0.5$$

$$\log_e e^{3k} = \log_e 0.5$$

$$3k = \ln 0.5$$

$$k = \frac{\ln 0.5}{3}$$

$$k = -0.231$$

b) $P = 100(1 - e^{-0.231t})$

$$= 100(1 - e^{-0.231 \times 5})$$

$$= 100 - 100e^{-1.155}$$

$$= 100 - 31.51$$

$$= 68.49$$

$$= 68.5$$

Question 12, (1) (1) (2) (2) (1)

a) i) (13, -4)

ii) $x^2 + y^2 + 14x - 22y + c = 0$

Substitute (13, -4) to give

$$13^2 + (-4)^2 + 14(13) - 22(-4) + c = 0$$

Simplify to give $c = -455$

b) i) C_1 radius = 10

$$C_2 \ x^2 + y^2 + 14x - 22y - 455 = 0$$

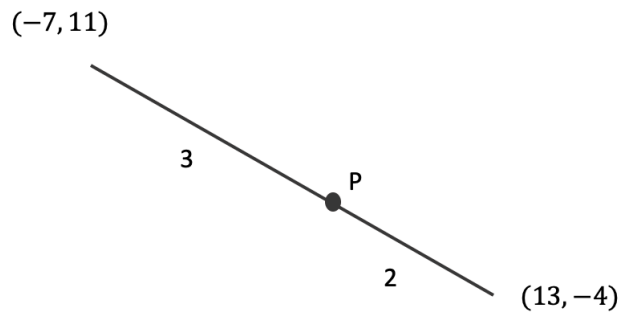
$$\text{Radius} = \sqrt{7^2 + 11^2 + 455}$$

$$= 25$$

$$\text{Ratio} = 15:10$$

ii) C_1 centre = $(13, -4)$

C_2 centre = $(-7, 11)$



Using the stepping out technique:

For the x co-ordinate, $\frac{3}{5} \times 20 = 12$, $-7 + 12 = 5$

For the y co-ordinate, $\frac{3}{5} \times 15 = 9$, $11 - 9 = 2$

$P = (5, 2)$

c) Radius = $25 + 15 = 40$

Centre = $(5, 2)$

$$(x - 5)^2 + (y - 2)^2 = 40^2$$

$$(x - 5)^2 + (y - 2)^2 = 1600$$