2018 Higher Paper 1

Question 1, (3)

Mid-point of PQ $= (1, 2)$

Let $s = (1, 2)$

$$m_{RS} = \frac{6-2}{3-1} = 2$$

Using $y - b = m(x-a)$ with $(1, 2)$ gives

$y - 2 = 2(x-1)$

$y - 2 = 2x - 2$

$y = 2x$

Question 2, (3)

$$g(x) = \frac{1}{5}x - 4$$

$$y = \frac{1}{5}x - 4$$

$5y = x - 20$

$x = 5y + 20$

$g^{-1}(x) = 5x + 20$

Question 3, (3)

$h(x) = 3\cos(2x)$

$h'(x) = -6\sin(2x)$

$$h'\left(\frac{\pi}{6}\right) = -6\sin\left(2\ x\ \frac{\pi}{6}\right)$$

$$= -6\sin\ (\frac{\pi}{3})$$

$$= -6\ x\ \frac{\sqrt{3}}{2}$$

$$= -3\sqrt{3}$$

Question 4, (4)

$x^2 + y^2 - 12x - 6y - 23 = 0$

Circle centre $= (6, 3)$

Let $C = (6, 3)$

$$m_{KC} = \frac{3-(-5)}{6-8} = \frac{8}{-2} = -4$$

$$m_{perp} = \frac{1}{4}$$

Using $y - b = m(x - a)$ with $(6, 3)$ gives

$y - 3 = \frac{1}{4}(x - 6)$

$4y - 12 = x - 6$

$4y = x + 6$

$y = \frac{1}{4}x + \frac{6}{4}$

Question 5, (1) (1)

a) $\frac{8}{10} = \frac{4}{5}$

b) $\frac{4}{5}$ of $(9 - 4)$

$= \frac{4}{5} x\, 5$

$= 4$

$t = 4$

Question 6, (3)

$\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{\frac{1}{3}}$

$= \log_5 250 - \log_5 2$

$= \log_5 \frac{250}{2}$

$= \log_5 125$

$= 3$

Question 7, (1) (3) (4)

a) $(0, 5)$

b) $y = x^3 - 3x^2 + 2x + 5$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$m_{tan} = 3(0)^2 - 6(0) + 2$

$= 2$

$y = 2x + 5$

c) $2x + 5 = x^3 - 3x^2 + 2x + 5$

$x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

$x = 0, x = 3$

When $x = 3, y = (2\ x\ 3) + 5$

$= 11$

$Q = (3, 11)$

Question 8, (2)

$y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

Gradient $= \sqrt{3}$

So, $\tan\theta = \sqrt{3}$

$\theta = 60°$ from exact values

Question 9, (1) (2)

a) $\overrightarrow{BC} = -\underline{t} + \underline{u} = \underline{u} - \underline{t}$

b) $\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CA} + \overrightarrow{AD}$

$$= \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AD}$$

$$= \frac{1}{2}(\underline{u} - \underline{t}) - \underline{u} + \underline{v}$$

$$= \frac{1}{2}\underline{u} - \frac{1}{2}\underline{v} - \underline{u} + \underline{v}$$

$$= \underline{v} - \frac{1}{2}\underline{u} - \frac{1}{2}\underline{t}$$

Question 10, (4)

$$\frac{dy}{dx} = 6x^2 - 3x + 4$$

$$\int 6x^2 - 3x + 4\ dx$$

$= 2x^3 - \frac{3}{2}x^2 + 4x + c$

So, $y = 2x^3 - \frac{3}{2}x^2 + 4x + c$

Substitute $x = 2, y = 14$ to give

$14 = 2(2^3) - \frac{3}{2}(2^2) + 4(2) + c$

Rearrange to give

$c = -4$

So, $y = 2x^3 - \frac{3}{2}x^2 + 4x - 4$

Question 11, (2) (3)

a) $y = 1 - \log_3 x$

$y = -\log_3 x + 1$

The original graph needs to be reflected in the x-axis and moved up 1 unit

b) $1 - \log_3 x = \log_3 x$

$1 = 2\log_3 x$

$\log_3 x = \frac{1}{2}$

$3^{\frac{1}{2}} = x$

$x = \sqrt{x}$

Question 12, (1) (3)

a) $\underline{a} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$$2\underline{a} + \underline{b} = 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$$

b) $\left|2\underline{a} + \underline{b}\right| = \sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$

$$36 + 9 + 16 + 8p + p^2 = 49$$

$$p^2 + 8p + 12 = 0$$

$$(p+6)(p+2) = 0$$

$$p = -6, p = -2$$

Question 13, (3) (1) (3)

a) i) $\sin(2x) = 2\sin x \cos x$

$$= 2 \times \frac{2}{\sqrt{11}} \times \frac{\sqrt{7}}{\sqrt{11}}$$

$$= \frac{4\sqrt{7}}{11}$$

ii) $\cos(2x) = 1 - 2sin^2x$

$$= 1 - 2(\frac{2}{\sqrt{11}})^2$$

$$= 1 - 2(\frac{4}{11})$$

$$= 1 - \frac{8}{11}$$

$$= \frac{3}{11}$$

b) $\sin(3x) = \sin(2x + x)$

$$= \sin(2x)\cos x + \cos(2x)\sin x$$

$$= \left(\frac{4\sqrt{7}}{11}\ x\ \frac{\sqrt{7}}{11}\right) + (\frac{3}{11}\ x\ \frac{2}{11})$$

$$= \frac{28}{11\sqrt{11}} + \frac{6}{11\sqrt{11}}$$

$$= \frac{34}{11\sqrt{11}}$$

Question 14, (5)

$$\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}} dx = \int_{-4}^{9} (2x+9)^{-\frac{2}{3}} dx$$

$$= [\frac{(2x\ +\ 9)^{\frac{1}{3}}}{\frac{1}{3}\ x\ 2}]_{-4}^{9}$$

$$= [\frac{3}{2}(2x+9)^{\frac{1}{3}}]_{-4}^{9}$$

$$= \left(\frac{3}{2}\left(27^{\frac{1}{3}}\right)\right) - (\frac{3}{2}\left(1^{\frac{1}{3}}\right))$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

Question 15, (5)

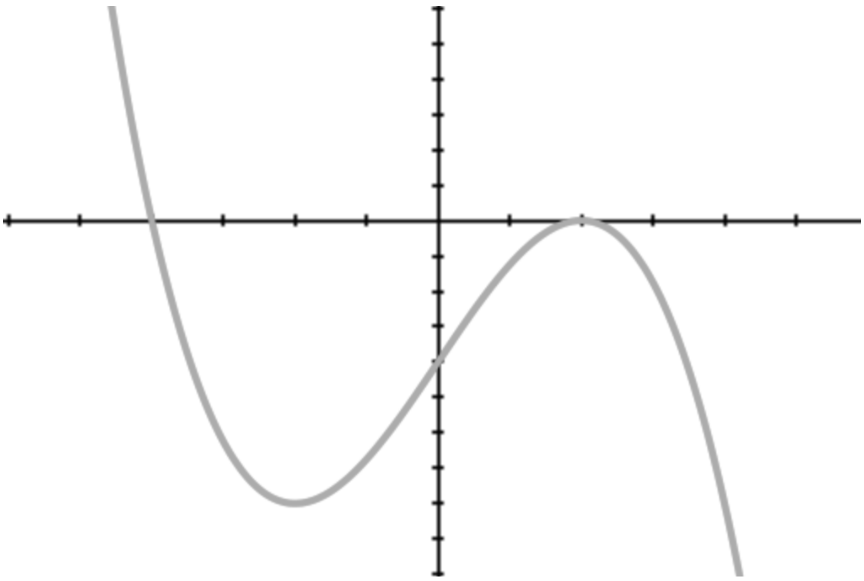
Since $(x+4)$ is a factor, $x = -4$ is a root

Since $x = 2$ is a repeated root, $(x-2)$ is a repeated factor

So, the equation of the function can be written $f(x) = k(x+4)(x-2)(x-2)$

Since $f'(-2) = 0$, the graph has a stationary point at $x = -2$

Considering all of the above information and assuming a k value (which cannot be determined from the information) a possible graph is:



2018 Higher Paper 2

Question 1, (4)

$$\int_{-1}^{3} 3 + 2x - x^2 \, dx$$

$$= [3x + x^2 - \frac{1}{3}x^3]_{-1}^{3}$$

$$= (9 + 9 - 9) - (-3 + 1 + \frac{1}{3})$$

$$= 11 - \frac{1}{3}$$

$$= \frac{33}{3} - \frac{1}{3}$$

$$= \frac{32}{3}$$

Area $= 10.67\, units^2$

Question 2, (1) (4)

a) $\underline{u} \bullet \underline{v} = 7 + 32 - 15 = 24$

b) $\left|\underline{u}\right| = \sqrt{1 + 16 + 9} = \sqrt{26}$

$\left|\underline{v}\right| = \sqrt{49 + 64 + 25} = \sqrt{138}$

$$\cos\theta = \frac{24}{\sqrt{26}\sqrt{138}}$$

$$\theta = cos^{-1}\left(\frac{24}{\sqrt{26}\sqrt{138}}\right)$$

$$= 66.38°$$

Question 3, (3)

$f(x) = x^3 - 7x - 6$

$f'(x) = 3x^2 - 7$

$f'(2) = 5$

Since $f'(2) > 0$, the function is increasing

Question 4, (3)

$$-3x^2 - 6x + 7 = -3\left(x^2 + 2x\right) + 7$$
$$= -3\left[(x+1)^2 - 1\right] + 7$$
$$= -3(x+1)^2 + 3 + 7$$
$$= -3(x+1)^2 + 10$$

Question 5, (3) (2) (2)

a) Mid-point $PQ = (6, 1)$

$$m_{PQ} = \frac{4 + 2}{3 - 9} = \frac{6}{-6} = -1$$

So, $m_{L1} = 1$

$$y - b = m(x - a)$$
$$y - 1 = x - 6$$
$$y = x - 5$$

b) $3y + x = 25$

$$3y = 25 - x$$

$$y = x - 5$$
$$3y = 3x - 15$$

Equating gives

$$3x - 15 = 25 - x$$
$$4x = 40$$
$$x = 10$$

When $x = 10, y = 10 - 5 = 5$

So, $C = \left(10, 5\right)$

c) $C = \left(10, 5\right)$

$P = (3, 4)$

Distance $= \sqrt{(10-3)^2 + (5-4)^2}$

$= \sqrt{50}$

$(x-10)^2 + (y-5)^2 = 50$

Question 6, (2) (1) (6)

a) i) $f\left(g(x)\right) = 3 + \cos(2x)$

ii) $g\left(f(x)\right) = 2(3 + \cos x)$

$= 6 + 2\cos x$

b) $3 + \cos(2x) = 6 + 2\cos x$

$\cos(2x) - 2\cos x - 3 = 0$

$2cos^2x - 1 - 2\cos x - 3 = 0$

$2cos^2x - 2\cos x - 4 = 0$

$cos^2x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0$

$\cos x = 2$

No solutions

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

Question 7, (2) (2) (1) (3) (1)

a) i)

2	2	-3	-3	2
		4	2	-2
	2	1	-1	0

Zero remainder shows that $(x-2)$ is a factor

ii) $2x^3 - 3x^2 - 3x + 2 = (x-2)(2x^2 + x - 1)$

$$= (x-2)(2x-1)(x+1)$$

b) $u_5 = 2a - 3$

$$u_6 = au_5 - 1$$
$$= a(2a-3) - 1$$
$$= 2a^2 - 3a - 1$$

$$u_7 = au_6 - 1$$
$$= a\left(2a^2 - 3a - 1\right) - 1$$
$$= 2a^3 - 3a^2 - a - 1$$

c) i) $2a - 3 = 2a^3 - 3a^2 - a - 1$

$2a^3 - 3a^2 - 3a + 2 = 0$

From a) ii) we have

$(a-2)(2a-1)(a+1) = 0$

$a = -1, \frac{1}{2}, 2$

But since there is a limit, $-1 < a < 1$

So, $a = \frac{1}{2}$

ii) Limit $= \dfrac{b}{1 - a} = \dfrac{-1}{1 - \frac{1}{2}} = -2$

Question 8, (4) (1) (2)

$a)\ 2\cos x - \sin x = k\cos(x - a)$

$= k\ cos\ x\cos a + k\sin x\sin a$

$= k\cos a\cos x + k\sin a\sin x$

$2 = k\cos a$

$-1 = k\sin a$

$k = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\tan a = -\dfrac{1}{2}$

$tan^{-1}\left(\dfrac{1}{2}\right) = 26.4°$

From CAST $a = 360 - 26.4 = 333.6°$

$2\cos x - \sin x = \sqrt{5}\cos(x - 333.6)°$

b) i) $6\cos x - 3\sin x = 3(2\cos x - \sin x)$

$= 3\sqrt{5}\cos(x - 333.6)°$

Considering the graph of this function gives a minimum value of $-3\sqrt{5}$

ii) $3\sqrt{5}\cos(x - 333.6)° = -3\sqrt{5}$

$\cos(x - 333.6)° = -1$

$x - 333.6 = 180$

$x = 180 + 333.6 = 513.4$

$x = 513.4 - 360 = 153.4°$

Question 9, (6)

$$P(x) = 2x + \frac{128}{x}$$

$$= 2x + 128x^{-1}$$

$$P'(x) = 2 - 128x^{-2} = 2 - \frac{128}{x^2}$$

For minimum values, $P'(x) = 0$

$$2 - \frac{128}{x^2} = 0$$

$$2x^2 - 128 = 0$$

$$x^2 = 64$$

$$x = -8, x = 8$$

Discard $x = -8$ since x cannot be negative

x	1	8	10
$P'(x)$	−	0	+
Shape	\	—	/

Substitute $x = 8$ to give

$$P(x) = (2\ x\ 8) + \frac{128}{8}$$

$$= 16 + 16$$

$$= 32$$

Question 10, (4)

$$x^2 + (m-3)x + m = 0$$

$$a = 1, b = m - 3, c = m$$

$$b^2 - 4ac = (m-3)^2 - 4(1)(m)$$
$$= m^2 - 6m + 9 - 4m$$
$$= m^2 - 10m + 9$$

Since there are two roots, $b^2 - 4ac > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

Consider the roots of $m^2 - 10m + 9$ which are at $m = 1, m = 9$

Consider the graph of $m^2 - 10m + 9$ giving

$m < 1, m > 9$

Question 11, (4) (2)

a) $P = 100(1 - e^{-kt})$

$50 = 100(1 - e^{-kt})$

$0.5 = 1 - e^{-kt}$

$e^{-kt} = 0.5$

$\log_e e^{3k} = \log_e 0.5$

$3k = \ln 0.5$

$$k = \frac{\ln 0.5}{3}$$

$k = -0.231$

b) $P = 100(1 - e^{-0.231t})$

$= 100(1 - e^{-0.231 \ x \ 5})$

$= 100 - 100e^{-1.155}$

$= 100 - 31.51$

$= 68.49$

$= 68.5$

Question 12, (1) (1) (2) (2) (1)

a) i) $(13, \ -4)$

ii) $x^2 + y^2 + 14x - 22y + c = 0$

Substitute $(13, \ -4)$ to give

$13^2 + \ (-4)^2 + 14(13) - 22(-4) + c = 0$

Simplify to give $c = -455$

b) i) C_1 radius $= 10$

$C_2 \ x^2 + y^2 + 14x - 22y - 455 = 0$

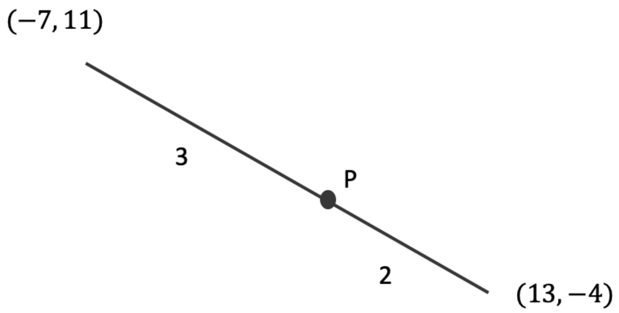
Radius $= \sqrt{7^2 + 11^2 + 455}$

$= 25$

Ratio $= 15 : 10$

ii) C_1 centre $= (13, \ -4)$

C_2 centre $= (-7, 11)$



Using the stepping out technique:

For the x co-ordinate, $\frac{3}{5}$ x $20 = 12, -7 + 12 = 5$

For the y co-ordinate, $\frac{3}{5}$ x $15 = 9, 11 - 9 = 2$

$P = (5, 2)$

c) Radius $= 25 + 15 = 40$

Centre $= (5, 2)$

$$(x - 5)^2 + (y - 2)^2 = 40^2$$
$$(x - 5)^2 + (y - 2)^2 = 1600$$