SQA National 5 Mathematics
Course Handbook

This handbook summarises the required knowledge for the SQA National 5 Mathematics course. The material contained in this guide should be considered necessary but not sufficient to be successful on the National 5 Mathematics course. In other words, you should strive to know and understand everything presented here but that knowledge should be supplemented with practice questions and exercises to gain a wider understanding of the topics and how they can be used.

In Mathematics **UNDERSTANDING** is the key. While it is important to learn formulas and techniques more important is to strive to gain insight into what a topic is really about. If you can achieve understanding then it becomes less important to memorise formulas and techniques because, in many cases, they follow easily from your understanding.

Calculations using Scientific Notation

If a number is very large or very small your calculator will present it in scientific notation otherwise it would not fit on the display.

Calculations can be performed on a scientific calculator by entering the numbers in scientific notation using the EXP or similar button.

Ex 1) Work out $(4.25 \ x \ 10^{6}) \ x \ (4 \ x \ 10^{-4})$

Enter this on your calculator as: $4.\ 2\ 5 \text{ EXP } 6 \ x \ 4 \text{ EXP } -4 \ =$

Giving 1700 as an ordinary number

Or $1.7 \ x \ 10^{3}$ in scientific notation

Ex 2) Give your answer to the following in scientific notation

$24{,}000{,}000 \ x \ 25{,}000{,}000$

Since the numbers are given as ordinary numbers you can enter them into the calculator directly

$24{,}000{,}000 \ x \ 25{,}000{,}000 \ = \ 6.0 \ x \ 10^{14}$

Some calculations involving scientific notation can be performed without using a calculator.

Ex 3) Work out $(5 \ x \ 10^{2}) \ + \ (3 \ x \ 10^{3})$

$5 \ x \ 10^{2} = 500$

$3 \ x \ 10^{3} = 3000$

$$(5 \ x \ 10^{2}) \ + \ (3 \ x \ 10^{3}) = 500 \ + \ 3000$$

$$= 3500$$

$$= 3.5 \ x \ 10^{3}$$

Ex 1) Write the following fraction with a rational denominator

$$\frac{3}{\sqrt{5}}$$

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \, x \, \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{5 \, x \, 5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$$

Ex 2) Write the following fraction with a rational denominator

$$\frac{\sqrt{6}}{\sqrt{2}}$$

$$\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} \, x \, \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{12}}{\sqrt{2 \, x \, 2}} = \frac{\sqrt{12}}{\sqrt{4}} = \frac{\sqrt{12}}{2}$$

In this case $\frac{\sqrt{12}}{2}$ can be simplified further as follows

$$\frac{\sqrt{12}}{2} = \frac{\sqrt{3 \, x \, 4}}{2} = \frac{\sqrt{3} \, x \, \sqrt{4}}{2} = \frac{\sqrt{3} \, x \, 2}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

Ex 3) Write the following fraction with a rational denominator

$$\frac{\sqrt{2}}{\sqrt{14}}$$

$$\frac{\sqrt{2}}{\sqrt{14}} = \frac{\sqrt{2}}{\sqrt{14}} \, x \, \frac{\sqrt{14}}{\sqrt{14}} = \frac{\sqrt{28}}{\sqrt{14 \, x \, 14}} = \frac{\sqrt{28}}{\sqrt{196}} = \frac{\sqrt{28}}{14}$$

$$\frac{\sqrt{28}}{14} = \frac{\sqrt{4 \, x \, 7}}{14} = \frac{\sqrt{4} \, x \, \sqrt{7}}{14} = \frac{2\sqrt{7}}{14} = \frac{\sqrt{7}}{7}$$

Algebraic Fractions

Algebraic Fractions are fractions where either the numerator or the denominator or both has a variable term, an unknown such as x.

Simplifying Algebraic Fractions

Algebraic Fractions are simplified by using a combination of factorising (either by using a common factor or factoring into two brackets) and cancelling terms.

Ex 1) Simplify the algebraic fraction

$$\frac{8x-4y}{2} = \frac{4(2x-y)}{2} = 2(2x-y)$$

$$\frac{12}{9-6x} = \frac{12}{3(3-2x)} = \frac{4}{(3-2x)}$$

$$\frac{5x+10}{15} = \frac{5(x+2)}{15} = \frac{(x+2)}{3}$$

$$\frac{ax+bx}{x} = \frac{x(a+b)}{x} = (a+b)$$

Ex 2) Simplify the algebraic fraction

$$\frac{x^2+3x}{x^2+4x+3} = \frac{x(x+3)}{(x+1)(x+3)} = \frac{x}{(x+1)}$$

$$\frac{x^2-1}{x-1} = \frac{x^2-1^2}{x-1} = \frac{(x-1)(x+1)}{x-1} = (x+1)$$

$$\frac{x^2-9}{2x^2-5x-3} = \frac{x^2-3^2}{(2x+1)(x-3)} = \frac{(x-3)(x+3)}{(2x+1)(x-3)} = \frac{(x+3)}{(2x+1)}$$

Straight Line Graphs

Straight line graphs are generated from *linear functions* (first order functions).

Straight line graphs are defined by their slope or steepness, measured by the *gradient* and how high up or down they are, measured by the *y-intercept*. The y-intercept is the point where the line intersects the y-axis.

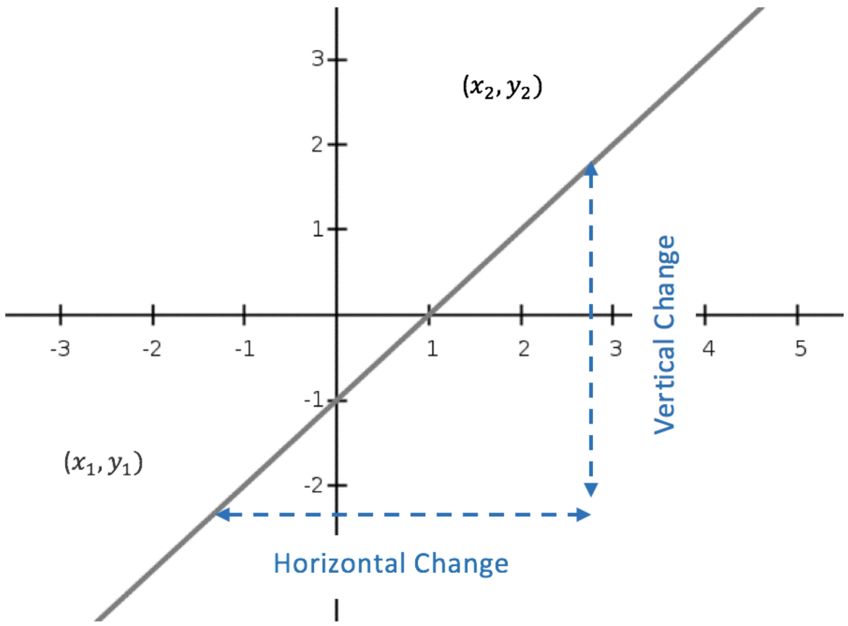
The Gradient of a Straight Line

The gradient, *m*, of a straight line can be calculated using the formula

$$\text{Gradient} = m = \frac{Vertical\ Change}{Horizontal\ Change}$$

In practice, we need two points on a line (x_1, y_1) and $(x_2,\ y_2)$ to find the gradient,

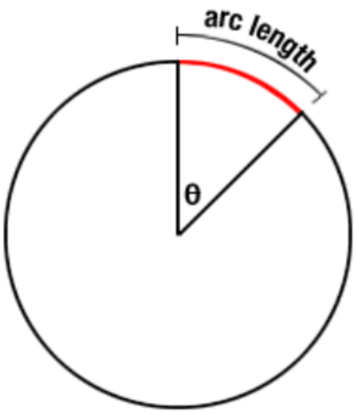
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{Vertical\ Change}{Horizontal\ Change}$$



Note that it doesn't matter which point you choose to be (x_1, y_1) and $(x_2,\ y_2)$.

Length of an Arc

An *arc* is part of the circumference of a circle. To find the length of an arc we find the circumference of the circle and then take only that part we are interested in.



Remember that the circumference $C = \pi d$, where d is the circle's diameter

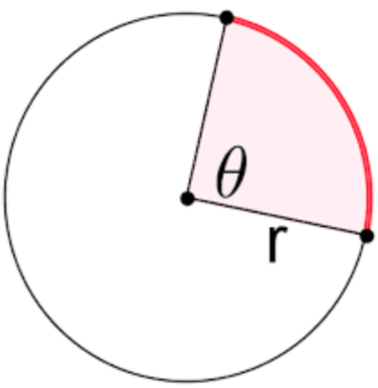
We determine the part of the circumference we're interested in by using an angle θ, as a ratio out of 360° since 360° represents the entire circumference.

Arc Length $= C \, x \, \dfrac{\theta}{360} = \pi d \, x \, \dfrac{\theta}{360}$

It may be helpful to remember this formula as '*angle out of 360 times the circumference*'

Area of a Sector

A *sector* is part of the area of a circle and looks like a 'slice' taken out of the circle. To find the area of a sector we need to find the area of the whole circle and then take only that part we are interested in.

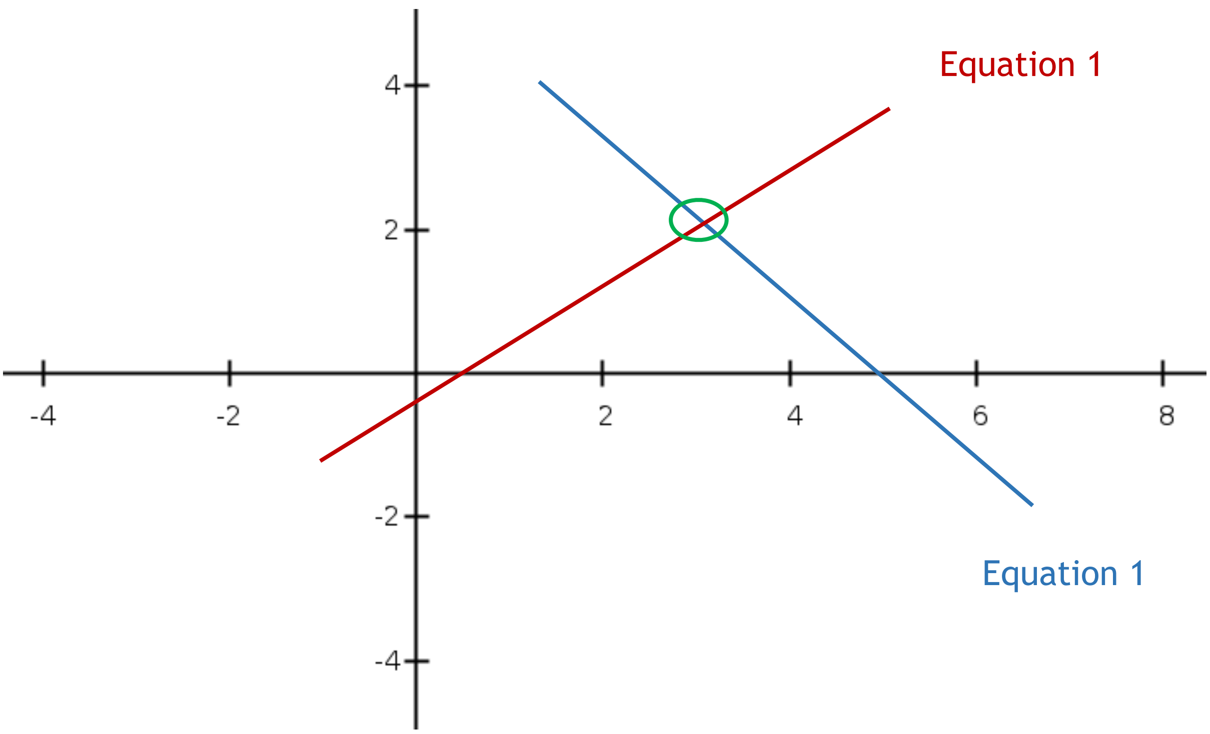


Simultaneous Equations

Simultaneous equations, sometimes called *systems of equations*, are such that the solutions must satisfy all of the equations in the system.

In general, we must have at least as many equations as you have variables in order to solve for them. In other words, if you have a system of equations containing two variables, x and y, then you need to have at least two equations with x and y in them in order to solve.

It's important to note that each equation in a system of equations represents a straight-line graph. If the solution $x,\ y$ satisfies both equations it must be a point lying on the graph of both – the only possible solution is therefore the point at which the two lines intersect, $(x,\ y)$.



In this example, the solution appears to be at $(3,\ 2)$ based on where the lines intersect.

Quadratic Functions

The features of quadratic function graphs that you should be familiar with are:

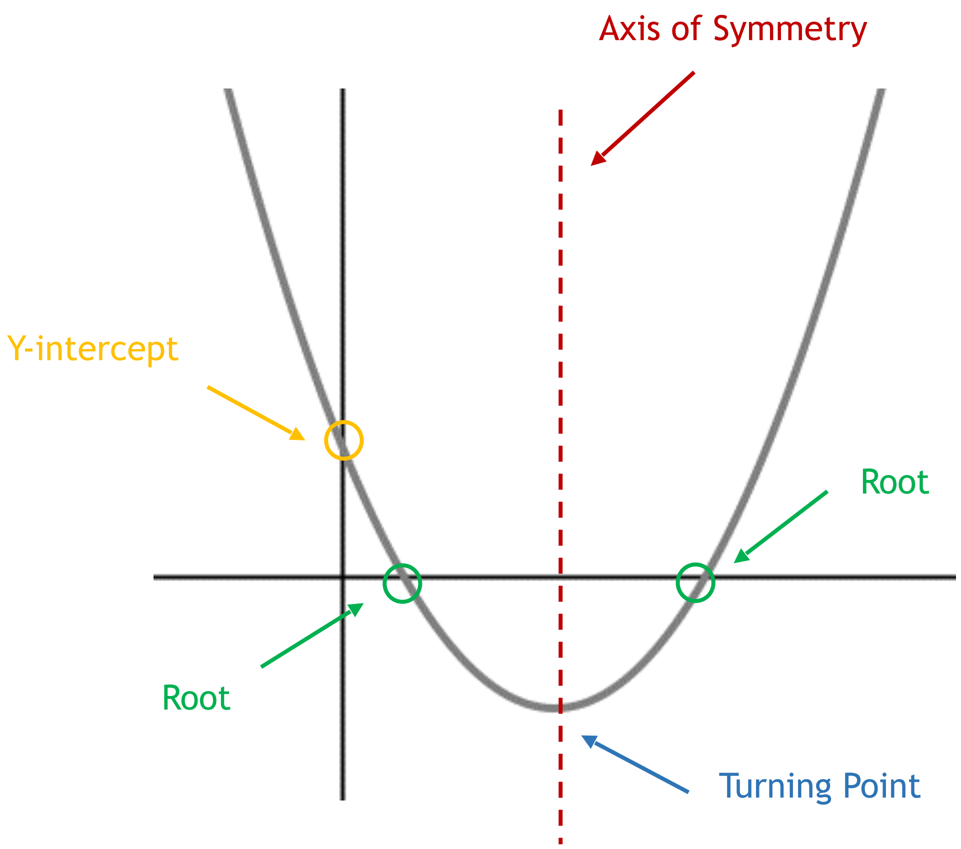
Roots: the point(s) where the graph crosses over the x-axis. There can be zero, one or two roots depending on the values of *a*, *b*, & *c*.

Turning point: the point at which the graph changes direction. Note this point can be at the top - called a *maximum turning point*, or at the bottom - called a *minimum turning point*.

Y-intercept: the point at which the graph crosses over the y-axis. There is only ever one y-intercept for a quadratic function.

Axis of Symmetry: the axis of symmetry is the line that splits the graph of a quadratic function down the middle.

Finding the co-ordinates of these points are what the main techniques of this section on quadratics focus on.



Using the Discriminant to Determine the Number of Roots

Sometimes we want to know the number of roots that a quadratic function has without having to work them out. We do this using part of the quadratic formula called the *discriminant*. The discriminant helps us 'discriminate' between the possible number of roots that a quadratic function could have. A quadratic function can have zero, one or two roots.

For the quadratic function $ax^2 + bx + c$ the discriminant is defined by $b^2 - 4ac$

If $b^2 - 4ac < 0$ the quadratic function has **zero** roots
If $b^2 - 4ac = 0$ the quadratic function has **one** real root
If $b^2 - 4ac > 0$ the quadratic function has **two** real roots

Note that one real root is sometimes referred to as *equal roots* or *repeated roots* since there are actually two roots but they are the same.

Ex 1) Determine the number of roots of the quadratic function $y = 2x^2 - 3x + 2$

$a = 2, b = -3, c = 2$

$b^2 - 4ac = (-3)^2 - 4x2x2 = 9 - 16 = -7$

Since $-7 < 0$ the quadratic $y = 2x^2 - 3x + 2$ has no real roots

Ex 2) Determine the number of roots of the quadratic function $y = x^2 + 5x - 7$

$a = 1, b = 5, c = -7$

$b^2 - 4ac = 5^2 - 4x1x(-7) = 25 + 28 = 53$

Since $53 > 0$ the quadratic $y = x^2 + 5x - 7$ has two real roots

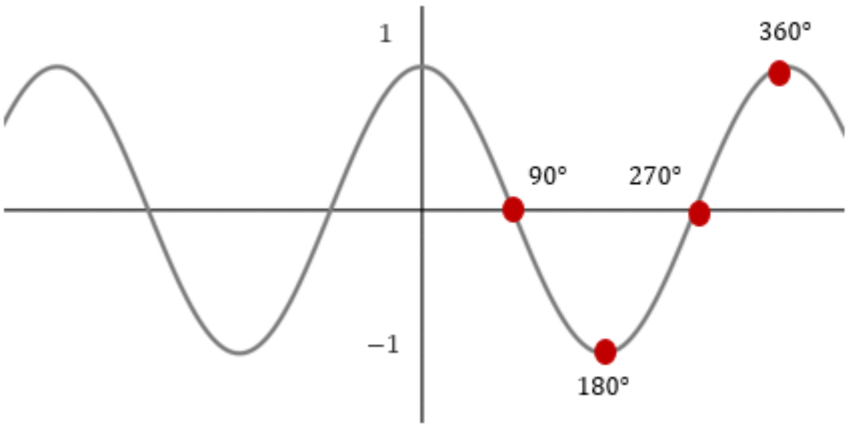
Ex 3) Determine the number of roots of the quadratic function $y = x^2 + 6x + 9$

$a = 1, b = 6, c = 9$

$b^2 - 4ac = 6^2 - 4x1x9 = 36 - 36 = 0$

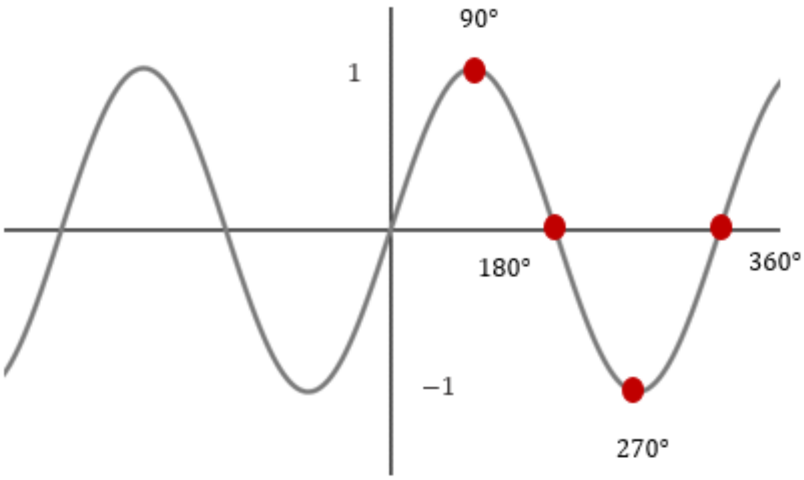
Since $b^2 - 4ac = 0$ the quadratic $y = x^2 + 6x + 9$ has one real root

The Cosine Function



The maximum value of $Cos\,x°$ is 1
The minimum value of $Cos\,x°$ is -1
The range of possible values of $Cos\,x°$ is between -1 and 1
1 is said to be the *amplitude* of $Cos\,x°$
The graph of $Cos\,x°$ repeats every $360°$ – $360°$ is said to be the *period* of $Cos\,x°$

The Sine Function



The maximum value of $Sin\,x°$ is 1
The minimum value of $Sin\,x°$ is -1
The range of possible values of $Sin\,x°$ is between -1 and 1
1 is said to be the *amplitude* of $Sin\,x°$
The graph of $Sin\,x°$ repeats every $360°$ – $360°$ is said to be the *period* of $Sin\,x°$

Standard Deviation

Standard deviation is similar to interquartile range in that it measures the spread of data within a data set. Like interquartile range, the standard deviation on its own doesn't tell us a great deal about the data but can be used to compare data sets.

Standard deviation measures how far, on average, the data points in a set of data are from the mean. The greater the deviation the more spread apart the data is, the smaller the deviation the more tightly packed the data. Notice that standard deviation uses the mean, not the median as a measure of average.

There are two formulae for standard deviation

$$s.d. = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$s.d. = \sqrt{\frac{\sum x^2 - (\sum x)^2 / n}{n - 1}}$$

Where n *is the number of data points in the set*

$\bar{x}$ *is the mean*

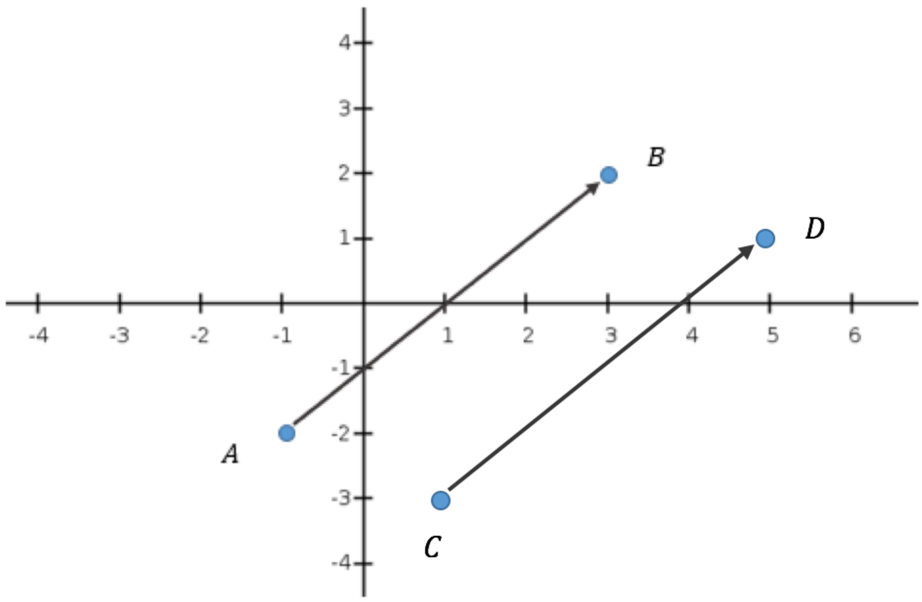
$\sum$ *means 'the sum of' indicating to add the values together*

Calculators with a statistical function are able to find the standard deviation but it's important to be able to work it out manually. A common approach to doing so is to construct a table which helps keep track of the calculation.

Vectors

The vector $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ is said to be the component form of $\overrightarrow{AB}$



Consider the vector $\overrightarrow{CD}$

$x - direction$; 1 to 3 is a difference of 4

$y - direction$; -3 to 1 is also a difference of 4

Giving vector $\overrightarrow{CD} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Notice that $\overrightarrow{AB} = \overrightarrow{CD}$.

In other words, vectors are not dependent upon their position in space since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are clearly in different positions but are the same vector. This reinforces the idea that vectors simply tell us how to get from one position to another.