Higher Mathematics

Trigonometric Identities - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 10, (2)

$$\begin{aligned}\cos(270 - a) &= \cos 270 \cos a + \sin 270 \sin a \\ &= 0 \bullet \cos a + (-1) \bullet \sin a \\ &= -\sin a\end{aligned}$$

2013 Paper 1 Question 23, (4) (2)

a) Let $\sqrt{3}\sin x - \cos x = k\sin(x - a) = k\sin x\cos a - k\cos x\sin a$

$= k\cos a\sin x - k\sin a\cos x$

By inspection we have $\sqrt{3} = k\cos a$ and $1 = k\sin a$

$$k = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\frac{k\sin a}{k\cos a} = \frac{1}{\sqrt{3}} = \tan a$$

Giving $a = 30°$ by exact values

Since we want a to be such that both $\sin a$ and $\cos a$ are positive, by CAST,

a must be in quadrant 1. So, $30°$ is correct

Thus $\sqrt{3}\sin x - \cos x = 2\sin(x - 30)°$

b) $4 + 5\cos x - 5\sqrt{3}\sin x = 4 - 5(\sqrt{3}\sin x - \cos x)$

$= 4 - 5 \bullet 2\sin(x - 30)$ from a)

$= 4 - 10\sin(x - 30)$

$= -10\sin(x - 30) + 4$

$-10\sin(x - 30)$ has a maximum value of 10

So, $y = -10\sin(x - 30) + 4$ has a maximum value of 14

<u>2014 Paper 1 Question 4, (2)</u>

Let $3\sin x - 4\cos x = k\cos(x - a)$

$= k\cos x\cos a + k\sin x\sin a$

$= k\cos a\cos x + k\sin a\sin x$

By inspection $3 = k\sin a$ and $-4 = k\cos a$

<u>2014 Paper 1 Question 7, (2)</u>

$\sin 2a = 2\sin a\cos a$

From SOHCAHTOA

$$\sin 2a = 2 \bullet \frac{3}{\sqrt{34}} \bullet \frac{5}{\sqrt{34}}$$

$$= \frac{30}{34} = \frac{15}{17}$$

<u>2014 Paper 1 Question 18, (2)</u>

$1 - 2\sin^2 15° = \cos(2 \bullet 15°)$ from the double angle formulas

$= \cos 30°$

$$= \frac{\sqrt{3}}{2}$$

2015 Paper 1 Question 10, (1) (2)

a) Using right-triangle with sides, 3,4, & 5 we have

$$\cos 2x = \frac{4}{5}$$

b) $\cos 2x = 2\cos^2 x - 1$

$$\frac{4}{5} = 2(\cos x)^2 - 1$$

$$\frac{9}{5} = 2(\cos x)^2$$

$$\frac{9}{10} = (\cos x)^2$$

$$\cos x = \sqrt{\frac{9}{10}}$$

2015 Paper 2 Question 9, (8)

Let $36\sin\left(1.5t\right) - 15\cos\left(1.5t\right) = k\sin(1.5t - a)$

$$= k\sin\left(1.5t\right)\cos a - k\cos\left(1.5t\right)\sin a$$

$$= k\cos a\sin\left(1.5t\right) - k\sin a\cos(1.5t)$$

By inspection we have $36 = k\cos a$ and $15 = k\sin a$

$$k = \sqrt{36^2 + 15^2} = 39$$

$$\frac{k\sin a}{k\cos a} = \frac{15}{36} = \tan a$$

$a = tan^{-1}\left(\frac{15}{36}\right) = 0.39$ radians (confirm this answer is in the correct quadrant using CAST)

So, $36\sin\left(1.5t\right) - 15\cos\left(1.5t\right) = 39\sin(1.5t - 0.39)$

$$h = 36\sin\left(1.5t\right) - 15\cos\left(1.5t\right) + 65$$

$= 39\sin(1.5t - 0.39) + 65$ (from above)

Let $h = 100$ to give

$100 = 39\sin(1.5t - 0.39) + 65$

$35 = 39\sin(1.5t - 0.39)$

$\frac{35}{39} = \sin(1.5t - 0.39)$

$1.5t - 0.39 = \sin^{-1}\left(\frac{35}{39}\right) = 1.11$ radians

$1.5t = 1.5,$

$t = 1$

Second solution for $\frac{35}{39} = \sin(1.5t - 0.39)$ is given by

$\pi - 1.11 = 2.03$ radians, so

$1.5t - 0.39 = 2.03$

$t = 1.61$

So, the solutions are $t = 1$ second and $t = 1.61$ seconds

2016 Paper 1 Question 13, (5)

$\cos(q - p) = \cos p \cos q + \sin q \sin p$

Use Pythagoras to work out the triangle hypotenuse

$\cos(q - p) = \left(\frac{4}{5} \bullet \frac{4}{\sqrt{17}}\right) + (\frac{3}{5} \bullet \frac{1}{\sqrt{17}})$

$= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}}$

$= \frac{19}{5\sqrt{17}}$

Multiply by $\frac{\sqrt{17}}{\sqrt{17}}$ to give

$$\cos\left(q - p\right) = \frac{19\sqrt{17}}{85}$$

2016 Paper 2 Question 8, (4) (4)

a) Let $5\cos x - 2\sin x = k\cos(x + a)$

$$= k\cos x\cos a - k\sin a\sin x$$

$$= k\cos a\cos x - k\sin a\sin x$$

By inspection $k\cos a = 5$ and $k\sin a = 2$

$$k = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$\frac{k\sin a}{k\cos a} = \frac{2}{5} = \tan a$$

$a = tan^{-1}\left(\frac{2}{5}\right) = 0.38$ radians

So, $5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.38)$

b) Equating the line and curve gives

$$10 + 5\cos x - 2\sin x = 12$$

$$5\cos x - 2\sin x = 2$$

Using the result from part a) we have

$$\sqrt{29}\cos(x + 0.38) = 2$$

$$\cos(x + 0.38) = \frac{2}{\sqrt{29}}$$

First solution

$$x + 0.38 = cos^{-1}(\frac{2}{\sqrt{29}})$$

$$x + 0.38 = 1.19$$

$x = 0.81$ radians

Second solution

$$x + 0.38 = 2\pi - 0.81$$

$x = 4.71$ radians

<u>2016 Paper 2 Question 11, (4) (2)</u>

a) $\sin 2x \tan x = 2\sin x \cos x \tan x$

$$= 2\sin x \cos x \frac{\sin x}{\cos x}$$

$$= 2\sin^2 x$$

$= 1 - 2\cos 2x$ from double angle formula

b) $f(x) = \sin 2x \tan x$

$$= 1 - 2\cos 2x$$

$$f'(x) = 4\sin 2x$$

<u>2017 Paper 1 Question 14, (4) (3)</u>

a) Let $\sqrt{3}\sin x - \cos x = k\sin(x - a)$

$$= k\sin x \cos a - k\cos x \sin a$$

$$= k\cos a \sin x - k\sin a \cos x$$

By inspection $\sqrt{3} = k\cos a$ and $1 = k\sin a$

$$k = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

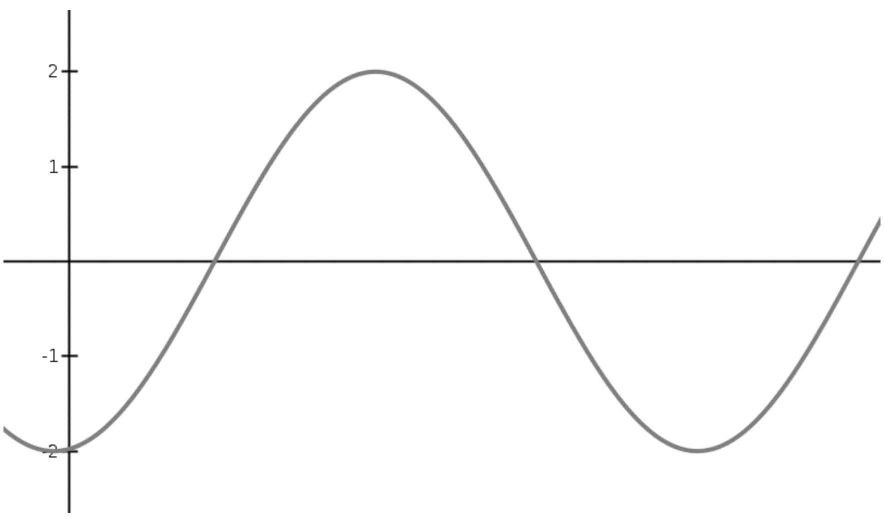
$$\frac{k\sin a}{k\cos a} = \frac{1}{\sqrt{3}} = \tan a$$

$$a = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Confirm that this answer is in the correct quadrant using CAST

So, $\sqrt{3}\sin x - \cos x = 2\sin(x - 30)°$

b) Since $\sqrt{3}\sin x - \cos x = 2\sin(x - 30)°$ the graph of $y = \sqrt{3}\sin x - \cos x$ is the same as the graph of $y = 2\sin(x - 30)$



2017 Paper 2 Question 11, (3) (3)

a) $\dfrac{\sin 2x}{2\cos x} - \sin x\cos^2 x = \dfrac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$

$= \sin x - \sin x\cos^2 x$

$= \sin x - \sin x(1 - \sin^2 x)$ since $\sin^2 x + \cos^2 x = 1$

$= \sin x - \sin x + \sin^3 x$

$= \sin^3 x$

b) $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{2\cos x} - \sin x\cos^2 x\right) = \dfrac{d}{dx}\left(\sin^3 x\right)$

$= \dfrac{d}{dx}(\sin x)^3$

$= 3(\sin x)^2 \bullet \cos x$

$= 3\cos x\sin^2 x$