

Higher Mathematics

Trigonometric Equations - Solutions - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 15, (2)

$$\tan \frac{\pi}{4} = 1 \text{ so } \frac{\pi}{4} \text{ is our reference angle}$$

Using CAST tan is negative in quadrants 2 & 4

$$\text{So, the solutions are } \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ and } \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

However, these solutions are for $\frac{x}{2}$ giving $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$

But since $\frac{5\pi}{2}$ is out of our range so $x = \frac{3\pi}{2}$

2013 Paper 2 Question 8, (6)

$$\sin 2x = 2 \cos^2 x$$

$$2 \sin x \cos x = 2 \cos^2 x$$

$$2 \sin x \cos x - 2 \cos^2 x = 0$$

$$2 \cos x (\sin x - \cos x) = 0$$

Separating into two equations gives

$$2 \cos x = 0$$

$$\sin x - \cos x = 0$$

$$\cos x = 0$$

$$\sin x = \cos x$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{\sin x}{\cos x} = 1$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

2014 Paper 2 Question 6, (5)

$$\sin x - 2\cos 2x = 1$$

Using the double angle formula $\cos 2x = 1 - 2\sin^2 x$ we have

$$\sin x - 2(1 - 2\sin^2 x) = 1$$

$$\sin x - 2 + 4\sin^2 x = 1$$

$$4\sin^2 x + \sin x - 3 = 0$$

$$(4\sin x - 3)(\sin x + 1) = 0$$

Separating into two equations gives

$$4\sin x - 3 = 0$$

$$\sin x + 1 = 0$$

$$\sin x = \frac{3}{4}$$

$$\sin x = -1$$

$$x = 0.8 \text{ rad}$$

$$x = \frac{3\pi}{2}$$

$$\text{So, } x = \pi - 0.8 = 2.3 \text{ rad}$$

2017 Paper 2 Question 6, (5)

$$5\sin x - 4 = 2\cos 2x$$

$$5\sin x - 4 = 2(1 - 2\sin^2 x) \text{ from double angle formula}$$

$$5\sin x - 4 = 2 - 4\sin^2 x$$

Rearranging gives

$$4\sin^2 x + 5\sin x - 6 = 0$$

$$(4\sin x - 3)(\sin x + 2) = 0$$

Separating into two equations gives

$$4\sin x - 3 = 0 \text{ and } \sin x + 2 = 0$$

$$4\sin x - 3 = 0$$

$$\sin x = \frac{3}{4}$$

$$x = \sin^{-1}\left(\frac{3}{4}\right) = 0.85 \text{ radians}$$

$$x = \pi - 0.85 = 2.3 \text{ radians}$$

$$\sin x + 2 = 0$$

$$\sin x = -2$$

No solutions since $-1 \leq \sin x \leq 1$