

Higher Mathematics

Straight Lines - Solutions - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 5, (2)

$$5x + 3y - 6 = 0$$

Rearranging gives

$$y = -\frac{5}{3}x + 2$$

Gradient of this line is $-\frac{5}{3}$

Since L is parallel it also has gradient $-\frac{5}{3}$

Using $y - b = m(x - a)$ with $(-2, -1)$ gives

$$y - (-1) = -\frac{5}{3}(x - (-2))$$

Rearranging gives

$$y = -\frac{5}{3}x - \frac{8}{3}$$

2013 Paper 2 Question 2, (3) (3) (3)

$$\text{a) } m_{PQ} = \frac{6 - 2}{5 - 7} = -2$$

$$m_{QR} = \frac{1}{2} \text{ since } PQ \text{ and } QR \text{ are perpendicular}$$

Using $y - b = m(x - a)$ with $Q = (5, 6)$ we have

$$y - 6 = \frac{1}{2}(x - 5)$$

$$y = \frac{1}{2}x + \frac{7}{2}$$

$$\text{b) } x + 3y = 13$$

$$y = -\frac{1}{3}x + \frac{13}{3}$$

Equating QR with PT gives

$$\frac{1}{2}x + \frac{7}{2} = -\frac{1}{3}x + \frac{13}{3}$$

Multiply through by 6 to simplify

$$3x + 21 = -2x + 26$$

$$x = 1$$

Substituting $x = 1$ into $y = \frac{1}{2}x + \frac{7}{2}$ gives

$$y = \frac{1}{2} + \frac{7}{2} = 4$$

$$T = (1, 4)$$

$$\text{c) } \vec{QT} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\text{So, } R = (1 - 4, 4 - 2) = (-3, 2)$$

$$\vec{QP} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\text{So, } S = (-3 + 2, 2 - 4) = (-1, -2)$$

2014 Paper 1 Question 2, (2)

$$m_{CT} = \frac{2 - (-1)}{1 - 3} = -\frac{3}{2}$$

$$m_{tan} = \frac{2}{3} \text{ since perpendicular}$$

Using $y - b = m(x - a)$ with $(3, -1)$ gives

$$y - (-1) = \frac{2}{3}(x - 3)$$

$$y = \frac{2}{3}x - 3$$

2014 Paper 2 Question 1, (4) (2) (2)

$$a) m_{AB} = \frac{2 - 0}{5 - 3} = 1$$

So, gradient of perpendicular line is -1

Midpoint of AB = $(4, 1)$

Using $y - b = m(x - a)$ with $(4, 1)$ we have

$$y - 1 = -1(x - 4)$$

$$y = -x + 5 \quad (1)$$

$$b) y + 2x = 6$$

$$y = -2x + 6 \quad (2)$$

Equating (1) & (2) gives

$$-x + 5 = -2x + 6$$

$$x = 1$$

When $x = 1, y = 4$

So, $T = (1, 4)$

c) $A = (3, 0), T = (1, 4)$

$$m_{AT} = \frac{4 - 0}{1 - 3} = -2$$

Using $y - b = m(x - a)$ with $(1, 4)$ gives

$$y - 4 = -2(x - 1)$$

Rearranging gives

$$y = -2x - 2$$

2015 Paper 1 Question 9, (3)

$$y + \sqrt{3}x = 0$$

$$y = -\sqrt{3}x$$

So, AB has a gradient of $-\sqrt{3}$ since parallel

$$\tan 150^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

So, BC has a gradient of $-\frac{\sqrt{3}}{3}$

Thus A, B, C cannot be collinear

2015 Paper 2 Question 1, (4) (3)

a) $m_{AB} = \frac{7 - (-5)}{-5 - (-1)} = -3$

$$m_{alt} = \frac{1}{3} \text{ since perpendicular to AB}$$

Using $y - b = m(x - a)$ with $(13, 3)$ gives

$$y - 3 = \frac{1}{3}(x - 13)$$

Rearranging gives

$$y = \frac{1}{3}x + \frac{26}{3}$$

b) Mid-point of AC = (4, 5) by inspection

$$m_{med} = \frac{5 - (-5)}{4 - (-1)} = 2$$

Using $y - b = m(x - a)$ with (4, 5) we have

$$y - 5 = 2(x - 4)$$

$$y = 2x - 3$$

c) Equating the two lines gives

$$\frac{1}{3}x + \frac{26}{3} = 2x - 3$$

$$x + 26 = 6x - 9$$

$$5x = 35$$

$$x = 7$$

$$\text{When } x = 7, y = 11$$

So, the point of intersection is (7, 11)

2016 Paper 1 Question 1, (2)

$$y + 4x = 7$$

$$y = -4x + 7$$

Any line parallel to this has gradient of -4

Using $y - b = m(x - a)$ with $(-2, 3)$ gives

$$y - 3 = -4(x - (-2))$$

$$y - 3 = -4(x + 2)$$

$$y = -4x - 5$$

2016 Paper 2 Question 1, (1) (2) (3)

a) i) $M = (2, 4)$

$$\text{ii) } m_{PM} = \frac{-4 - 4}{0 - 2} = 4$$

Using $y - b = m(x - a)$ with $(2, 4)$ gives

$$y - 4 = 4(x - 2)$$

$$y = 4x - 4$$

$$\text{b) } m_{PR} = \frac{6 - (-4)}{10 - 0} = 1$$

$$\text{So, } m_{\text{perp}} = -1$$

Using $y - b = m(x - a)$ with $(2, 4)$ gives

$$y - 4 = -1(x - 2)$$

$$y = -x + 6$$

c) Midpoint of PR = $(5, 1)$

Test whether $(5, 1)$ lies on L

Substituting $x = 5$ into L gives

$$y = -5 + 6 = 1$$

So, $(5, 1)$ does lie on the line L

2017 Paper 1 Question 7, (3)

Midpoint of AB = (2, 7)

Let M = (2, 7)

$$m_{CM} = \frac{11 - 7}{2 - 2} = \text{undefined}$$

So, the line is vertical and since it goes through (2, 7) has equation

$$x = 2$$

2017 Paper 1 Question 11, (3)

$$3y - 2x = 4$$

$$y = \frac{2}{3}x + \frac{4}{3}$$

The gradient of this line is $\frac{2}{3}$

The gradient of the line joining A&B is also $\frac{2}{3}$ since parallel

$$m_{AB} = \frac{2 - a}{-7 - 5} = \frac{2}{3}$$

$$\frac{2 - a}{-12} = \frac{2}{3}$$

$$2 - a = -8$$

$$a = 10$$

2017 Paper 2 Question 1, (4) (2) (2)

$$\text{a) } m_{BC} = \frac{0 - (-2)}{3 - 9} = -\frac{1}{3}$$

$$m_{\text{perp}} = 3$$

Midpoint of BC = (6, -1)

Using $y - b = m(x - a)$ with (6, -1) gives

$$y - (-1) = 3(x - 1)$$

$$y = 3x - 19$$

b) $m_{AB} = \tan 45^\circ = 1$

Using $y - b = m(x - a)$ with $(3, 0)$ gives

$$y - 0 = (x - 3)$$

$$y = x - 3$$

c) Equating gives $3x - 19 = x - 3$

$$2x = 16$$

$$x = 8$$

When $x = 8$, $y = 5$

So, the point of intersection is $(8, 5)$