Higher Mathematics

Recurrence Relations - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 8, (2)

$u_{n+1} = 0.1u_n + 8$

$u_1 = 0.1u_0 + 8$

$11 = 0.1u_0 + 8$

$3 = 0.1u_0$

$30 = u_0$

Since $-1 < 0.1 < 1$ the sequence does have a limit as n $\rightarrow \infty$

The answer is C) Only statement 2) is correct

2013 Paper 2 Question 1, (4)

$u_1 = 4, u_2 = 7, u_3 = 16$

$u_{n+1} = mu_n + c$

Substituting u_1 and u_2 gives

$u_2 = mu_1 + c$

$7 = 4m + c$

Substituting u_2 and u_3 gives

$u_3 = mu_2 + c$

$16 = 7m + c$

Now we have simultaneous equations to solve for m c

$7 = 4m + c$ (1)

$16 = 7m + c$ (2)

(2) - (1) gives

$9 = 3m$

$m = 3$

Substituting $m = 3$ into (2) gives

$16 = 7 \bullet 3 + c$

$c = -5$

So, $m = 3, c = -5$

<u>2014 Paper 1 Question 1, (2)</u>

$u_{n+1} = \frac{1}{3}u_n + 1$

$u_3 = \frac{1}{3}u_2 + 1 = \frac{1}{3} \bullet 15 + 1 = 6$

$u_4 = \frac{1}{3}u_3 + 1 = \frac{1}{3} \bullet 6 + 1 = 3$

<u>2014 Paper 1 Question 10, (2)</u>

For $u_{n+1} = au_n + b$ the limit occurs where $-1 < a < 1$

So, limit occurs where $-1 < (k-2) < 1$

Adding 2 to both sides gives $1 < k < 3$

<u>2015 Paper 2 Question 3, (1) (5)</u>

a) $t_2 = \frac{3}{4}t_1 + 13$

$= \frac{3}{4} \bullet 13 + 13$

$= \frac{39}{4} + \frac{52}{4}$

$= \frac{91}{4}$

b) $f_{n+1} = \frac{1}{3}f_n + 32$

Limit $= \dfrac{32}{1-\frac{1}{3}} = \dfrac{32}{\frac{2}{3}} = 48$

Since $48 < 50$ the frog does not escape from the well

$t_{n+1} = \dfrac{3}{4}t_n + 13$

Limit $= \dfrac{13}{1-\frac{3}{4}} = \dfrac{13}{\frac{1}{4}} = 52$

Since $52 > 50$ the toad does escape from the well

<u>2016 Paper 1 Question 3, (1) (1) (2)</u>

a) $u_{n+1} = \dfrac{1}{3}u_n + 10$

$u_4 = \dfrac{1}{3}u_3 + 10$

$u_4 = \dfrac{1}{3} \bullet 6 + 10$

$u_4 = 12$

b) Since $-1 < \dfrac{1}{3} < 1$ the sequence has a limit as $n \to \infty$

c) Limit $= \dfrac{10}{1-\frac{1}{3}} = \dfrac{10}{\frac{2}{3}} = 15$

2017 Paper 1 Question 9, (2) (1)

a) $u_{n+1} = mu_n + 6$

$u_2 = mu_1 + 6$

$13 = 28m + 6$

$7 = 28m$

$m = \frac{1}{4}$

b) i) Since $-1 < \frac{1}{4} < 1$ the sequence has a limit as $n \to \infty$

ii) Limit $= \frac{6}{1 - \frac{1}{4}} = \frac{6}{\frac{3}{4}} = 8$

2017 Paper 2 Question 8, (2) (4)

a) $u_{n+1} = ku_n - 20$

$u_1 = k \bullet u_0 - 20 = 5k - 20$

$u_2 = k \bullet u_1 - 20 = k(5k - 20) - 20$

$= 5k^2 - 20k - 20$

b) Let $u_2 < u_0$ to give

$5k^2 - 20k - 20 < 5$

$5k^2 - 20k - 25 < 0$

Let $5k^2 - 20k - 25 = 0$

$k^2 - 4k - 5 = 0$

$(k - 5)(k + 1) = 0$

$k = -1, k = 5$

So, $5k^2 - 20k - 25 < 0$ when $-1 \le k \le 5$