

## Higher Mathematics

### Recurrence Relations - Questions - 2013-2017

Marks are indicated in brackets after each question number

#### 2013 Paper 1 Question 8, (2)

A sequence is defined by the recurrence relation  $u_{n+1} = 0.1u_n + 8$ , with  $u_1 = 11$ .

Here are two statements about this sequence:

- (1)  $u_0 = 9.1$ ;
- (2) The sequence has a limit as  $n \rightarrow \infty$ .

Which of the following is true?

- A Neither statement is correct.
- B Only statement (1) is correct.
- C Only statement (2) is correct.
- D Both statements are correct.

#### 2013 Paper 2 Question 1, (4)

The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of  $m$  and  $c$ .

#### 2014 Paper 1 Question 1, (2)

A sequence is defined by the recurrence relation  $u_{n+1} = \frac{1}{3}u_n + 1$ , with  $u_2 = 15$ .

What is the value of  $u_4$ ?

### 2014 Paper 1 Question 10, (2)

A sequence is defined by the recurrence relation

$$u_{n+1} = (k - 2)u_n + 5 \quad \text{with } u_0 = 3.$$

For what values of  $k$  does this sequence have a limit as  $n \rightarrow \infty$ ?

### 2015 Paper 2 Question 3, (1) (5)

A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down  $\frac{2}{3}$  of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down  $\frac{1}{4}$  of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

$$\bullet \quad f_{n+1} = \frac{1}{3}f_n + 32, \quad f_1 = 32$$

$$\bullet \quad t_{n+1} = \frac{3}{4}t_n + 13, \quad t_1 = 13$$

where  $f_n$  and  $t_n$  are the heights reached by the frog and the toad at the end of the  $n$ th day after falling in.

(a) Calculate  $t_2$ , the height of the toad at the end of the second day.

(b) Determine whether or not either of them will eventually escape from the well.

### 2016 Paper 1 Question 3, (1) (1) (2)

A sequence is defined by the recurrence relation  $u_{n+1} = \frac{1}{3}u_n + 10$  with  $u_3 = 6$ .

(a) Find the value of  $u_4$ .

(b) Explain why this sequence approaches a limit as  $n \rightarrow \infty$ .

(c) Calculate this limit.

2017 Paper 1 Question 9, (2) (1)

A sequence is generated by the recurrence relation  $u_{n+1} = mu_n + 6$  where  $m$  is a constant.

- (a) Given  $u_1 = 28$  and  $u_2 = 13$ , find the value of  $m$ .
- (b) (i) Explain why this sequence approaches a limit as  $n \rightarrow \infty$ .  
(ii) Calculate this limit.

2017 Paper 2 Question 8, (2) (4)

Sequences may be generated by recurrence relations of the form

$$u_{n+1} = ku_n - 20, u_0 = 5 \text{ where } k \in \mathbb{R}.$$

- (a) Show that  $u_2 = 5k^2 - 20k - 20$ .
- (b) Determine the range of values of  $k$  for which  $u_2 < u_0$ .