Higher Mathematics

Quadratics - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 3, (2)

$a = 2, b = 4, c = 5$

$$
\begin{aligned}
b^2 - 4ac &= 4^2 - 4 \bullet 2 \bullet 5 \\
&= 16 - 40 \\
&= -24
\end{aligned}
$$

2013 Paper 1 Question 19, (2)

$1 - 2x - 3x^2 > 0$

Multiplying through by -1 gives

$3x^2 + 2x - 1 < 0$

$(3x - 1)(x + 1) < 0$

Consider $(3x - 1)(x + 1) = 0$

$x = -1, x = \frac{1}{3}$

These are the roots of the quadratic. By considering the graph we see it is negative (i.e. $y < 0$) for

$-1 < x < \frac{1}{3}$

So, the solutions are all x such that $-1 < x < \frac{1}{3}$

2013 Paper 1 Question 21, (3)

$2x^2 + 12x + 1$

$= 2\left(x^2 + 6x\right) + 1$

Now complete the square on the inside of the bracket

$= 2\left[(x+3)^2 - 9\right] + 1$

$= 2(x+3)^2 - 18 + 1$

$= 2(x+3)^2 - 17$

2014 Paper 1 Question 17, (2)

$3x^2 + 12x + 17 = 3\left(x^2 + 4x\right) + 17$

$= 3\left[(x+2)^2 - 4\right] + 17$

$= 3(x+2)^2 - 12 + 17$

$= 3(x+2)^2 + 5$

2015 Paper 1 Question 8, (4)

Area $= x(x-2) < 15$

$x^2 - 2x - 15 < 0$

$(x-5)(x+3) < 0$

Consider $\left(x-5\right)(x+3) = 0$

$x = -3, x = 5$

Since the graph of $x^2 - 2x - 15$ is U-shaped with roots at $x = -3, x = 5$ we have

$(x-5)(x+3) < 0$ for $-3 < x < 5$

2015 Paper 1 Question 11, (4) (6)

a) Circle centre $= (-8, -2)$, radius $= \sqrt{45}$

Let circle centre be C

$$m_{CT} = \frac{-2-(-5)}{-8-(-2)} = -\frac{1}{2}$$

$m_{tan} = 2$ since the tangent is perpendicular to CT

Using $y - b = m(x-a)$ with $(-2, -5)$ gives

$$y-(-5) = 2(x-(-2))$$
$$y+5 = 2x+4$$
$$y = 2x-1$$

b) Consider where the line, $y = 2x - 1$, and the parabola meet

To find this point equate them to give

$$2x - 1 = -2x^2 + px + 1 - p$$
$$2x^2 + 2x - px + p - 2 = 0$$

Grouping terms gives

$$2x^2 + (2-p)x + (p-2) = 0$$

The solutions to this equation are the points where the line and parabola meet.

Since the line is a tangent to the parabola they meet at only one point.

$$a = 2, b = (2-p), \ c = (p-2)$$

Since there is one solution $b^2 - 4ac = 0$

$$b^2 - 4ac = (2-p)^2 - 4 \bullet 2 \bullet (p-2) = 0$$
$$4 - 4p + p^2 - 8p + 16 = 0$$
$$p^2 - 12p + 20 = 0$$
$$(p-10)(p-2) = 0$$
$$p = 2, p = 10$$

But since $p > 3$ the solution is $p > 10$

<u>2016 Paper 1 Question 12, (2) (3)</u>

a) $h(x) = f\left(g(x)\right) = f(3 - x) = 2(3 - x)^2 - 4(3 - x) + 5 = 2x^2 - 8x + 11$

b) $h(x) = 2x^2 - 8x + 11 = 2\left(x^2 - 4x\right) + 11$

$$= 2\left[(x - 2)^2 - 4\right] + 11$$

$$= 2(x - 2)^2 - 8 + 11$$

$$= 2(x - 2)^2 + 3$$

<u>2016 Paper 2 Question 2, (3)</u>

$x^2 - 2x + 3 - p = 0$

$a = 1, b = -2, c = 3 - p$

For no real roots $b^2 - 4ac < 0$

$b^2 - 4ac < 0$

$(-2)^2 - 4 \bullet 1 \bullet (3 - p) < 0$

$4 - 12 + 4p < 0$

$4p < 8$

$p < 2$

<u>2017 Paper 1 Question 4, (3)</u>

$x^2 + 4x + \left(k - 5\right) = 0$

$a = 1, b = 4, c = k - 5$

For equal roots $b^2 - 4ac = 0$

$4^2 - 4 \bullet 1 \bullet \left(k - 5\right) = 0$

$$16 - 4k + 20 = 0$$

$$4k = 36, k = 9$$

2017 Paper 2 Question 4, (3) (2) (2)

a) $3x^2 + 24x + 50 = 3\left(x^2 + 8x\right) + 50$

$$= 3\left[(x+4)^2 - 16\right] + 50$$

$$= 3(x+4)^2 - 48 + 50$$

$$= 3(x+4)^2 + 2$$

b) $f(x) = x^3 + 12x^2 + 50x - 11$

$f'(x) = 3x^2 + 24x + 50$

c) $f'(x) = 3x^2 + 24x + 50$

$= 3(x+4)^2 + 2$ from a)

Since $(x+4)^2 \geq 0, 3(x+4)^2 + 2 > 0$

So, $f'(x) > 0$ meaning that $f(x)$ is strictly increasing for all x