Higher Mathematics

Polynomials - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 6, (2)

Let $f(x) = x^3 + 3x^2 - 5x - 6$

$f(2) = 2^3 + 3 \bullet 2^2 - 5 \bullet 2 - 6 = 4$

The remainder is 4 when $f(x)$ is divided by $(x - 2)$

2013 Paper 1 Question 17, (2)

$y = kx(x + a)^2$

Rewriting this gives

$y = k(x - 0)(x + a)(x + a)$

This shows that the roots are at $x = 0, x = -a$

So, by inspection, $a = 2$

So $y = kx(x + 2)^2$

Substituting the point (1, 3) gives

$3 = k(1 + 2)^2$

$3 = 9k$

$k = \frac{1}{3}$

2013 Paper 2 Question 3, (4) (5)

a) Let $f(x) = x^3 + 3x^2 + x - 5$

Using synthetic division gives

1	1	3	1	-5
		1	4	5
	1	4	5	0

$f(x) = (x-1)(x^2 + 4x + 5)$

Note that $x^2 + 4x + 5$ does not factorise

b) Let $g(x) = x^4 + 4x^3 + 2x^2 - 20x + 3$

$g'(x) = 4x^3 + 12x^2 + 4x - 20$

Stationary Points occur where $g'(x) = 0$ giving

$4x^3 + 12x^2 + 4x - 20 = 0$

$x^3 + 3x^2 + x - 5 = 0$

$(x-1)\left(x^2 + 4x + 5\right) = 0$ from a)

The solution to this equation is $x = 1$ since $x^2 + 4x + 5$ has no solutions

$(b^2 - 4ac = -4 < 0)$

Thus $g(x)$ has only one stationary point

2014 Paper 1 Question 15, (2)

Since the roots are at $x = -1, x = 2$ we have $y = k(x+1)(x-2)^2$ for some value k

Substituting the point $(0, -8)$ gives

$-8 = k(0+1)(0-2)^2$

$-8 = 4k$

$k = -2$

So, $y = -2(x+1)(x-2)^2$

2014 Paper 1 Question 22, (4) (3)

a) Let $f(x) = 6x^3 + 7x^2 + ax + b$

Since $x+1$ is a factor $f(-1) = 0$

$f(-1) = -6 + 7 - a + b = 0$

Rearranging gives $a = b + 1 \quad (1)$

Since 72 is the remainder when $f(x)$ is divided by $x-2$ we have

$f(2) = 48 + 28 + 2a + b = 72$

Rearranging gives $2a = -b - 4 \quad (2)$

Substituting (1) into (2) gives

$2\left(b+1\right) = -b - 4$

$2b + 2 = -b - 4$

$3b = -6$

$b = -2$

Using (1) we have

$a = -2 + 1 = -1$

b) $f(x) = 6x^3 + 7x^2 - x - 2$

Since $x+1$ is a factor we have

-1	6	7	-1	-2
		-6	-1	2
	6	1	-2	0

So, $f(x) = (x+1)(6x^2 + x - 2)$

$= (x+1)(3x+2)(2x-1)$

2015 Paper 1 Question 3, (4)

Let $f(x) = x^3 - 3x^2 - 10x + 24$

Using synthetic division gives

-3	1	-3	-10	24
		-3	18	-24
	1	-6	8	0

Since the remainder is zero, $(x+3)$ is a factor of $f(x)$

$$f(x) = (x+3)(x^2 - 6x + 8)$$
$$= (x+3)(x-4)(x-2)$$

2016 Paper 1 Question 15, (3) (1)

a) $f(x) = k(x-a)(x-b)^2$

By inspection of the roots we have

$$f(x) = k(x-4)(x+5)^2$$

Substituting the point (1, 9) gives

$$9 = k(1-4)(1+5)^2$$

Rearranging and solving for k gives

$$k = -\frac{1}{12}$$

b) $g(x) = f(x) - d$

Since d is positive the graph of $g(x)$ is the graph of $f(x)$ moved down by $'d'$ units

By considering the graph of $f(x)$ it must be moved down by at least 9 units to give only one root – in other words to move the point (1, 9) below the x axis.

So, $d > 9$

2016 Paper 2 Question 3, (2) (3) (1) (4)

a) i) Let $f(x) = 2x^3 - 9x^2 + 3x + 14$

Using synthetic division gives

-1	2	-9	3	14
		-2	11	-14
	2	-11	14	0

Since the remainder is zero $(x+1)$ is a factor of $f(x)$

ii) $2x^3 - 9x^2 + 3x + 14 = 0$

$$(x+1)\left(2x^2 - 11x + 14\right) = 0$$

$$(x+1)(2x-7)(x-2) = 0$$

$$x = -1, x = 2, x = \frac{7}{2}$$

b) i) $y = 2x^3 - 9x^2 + 3x + 14$

$$y = (x+1)(2x-7)(x-2)$$

By inspection of the roots $A = (-1, 0),\ B = (2, 0)$

ii) Area $= \int_{-1}^{2} \left(2x^3 - 9x^2 + 3x + 14\right) dx$

$$= [\frac{2}{4}x^4 - \frac{9}{3}x^3 + \frac{3}{2}x^2 + 14x]_{-1}^{2}$$

$$= [\frac{1}{2}x^4 - 3x^3 + \frac{3}{2}x^2 + 14x]_{-1}^{2}$$

$$= \left(8 - 24 + 6 + 28\right) - (\frac{1}{2} + 3 + \frac{3}{2} - 14)$$

$$= 27$$

Area $= 27\ units^2$

2017 Paper 2 Question 2, (2) (3)

a) $f(x) = 2x^3 - 5x^2 + x + 2$

Using synthetic division gives

1	2	-5	1	2
		2	-3	-2
	2	-3	-2	0

Since the remainder is zero, $x - 1$ is a factor of $f(x)$

b) $f(x) = 2x^3 - 5x^2 + x + 2$

$= (x - 1)(2x^2 - 3x - 2)$

$= (x - 1)(2x + 1)(x - 2)$

For $f(x) = 0$ we have

$(x - 1)(2x + 1)(x - 2) = 0$

$x = -\frac{1}{2},\ x = 1, x = 2$