

Higher Mathematics

Optimization - Solutions - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 2 Question 7, (3) (7)

a) $L = 3x + 4y$ by inspection of the diagram

$$\text{Area} = 24 = 2xy$$

$$y = \frac{24}{2x} = \frac{12}{x}$$

Substituting $y = \frac{12}{x}$ into L gives

$$L = 3x + 4 \cdot \frac{12}{x} = 3x + \frac{48}{x}$$

b) i) $L(x) = 3x + \frac{48}{x}$

$$= 3x + 48x^{-1}$$

$$L'(x) = 3 - 48x^{-2}$$

$$= 3 - \frac{48}{x^2}$$

Minimum values occur where $L'(x) = 0$

$$3 - \frac{48}{x^2} = 0$$




$$3x^2 - 48 = 0$$

$$x^2 - 16 = 0$$

$$(x + 4)(x - 4) = 0$$

$$x = -4, x = 4$$

Since x is a length it cannot be negative, so $x = 4$

x	1	4	10
$L'(x)$	+	0	-
<i>Shape</i>			

From the table we see that $x = 4$ gives a minimum value of $L(x)$

ii) Minimum length occurs where $x = 4$

$$L(4) = 3 \cdot 4 + \frac{48}{4} = 24m$$

$$\text{Minimum cost} = 24 \cdot \text{£}8.25 = \text{£}198$$

2015 Paper 2 Question 8, (1) (1) (8)

a) $T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$

i) If the crocodile does not travel on land then $x = 20$

$$\begin{aligned} \text{So, } T(20) &= 5\sqrt{36 + 20^2} + 4(20 - 20) \\ &= 5\sqrt{436} \cong 104 \text{ tenths of a second, } 10.4 \text{ seconds} \end{aligned}$$

ii) If the crocodile swims the shortest distance then $x = 0$

$$\begin{aligned} \text{So, } T(0) &= 5\sqrt{36 + 0^2} + 4(20 - 0) \\ &= 110 \text{ tenths of a second, } 11 \text{ seconds} \end{aligned}$$

b) $T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$

$$= 5(36 + x^2)^{\frac{1}{2}} + 80 - 4x$$

$$T'(x) = \left[\frac{5}{2}(36 + x^2)^{-\frac{1}{2}} \cdot 2x \right] - 4$$

$$= 5x(36 + x^2)^{-\frac{1}{2}} - 4$$

$$= \frac{5x}{(36 + x^2)^{\frac{1}{2}}} - 4$$

Minimum values occur where $T'(x) = 0$

$$\text{Let } \frac{5x}{(36 + x^2)^{\frac{1}{2}}} - 4 = 0$$

$$\frac{5x}{(36 + x^2)^{\frac{1}{2}}} = 4$$

$$4(36 + x^2)^{\frac{1}{2}} = 5x$$

$$(36 + x^2)^{\frac{1}{2}} = \frac{5x}{4}$$

Squaring both sides gives

$$36 + x^2 = \frac{25x^2}{16}$$

$$576 + 16x^2 = 25x^2$$




$$9x^2 - 576 = 0$$

$$x^2 - 64 = 0$$

$$x = 8, x = -8$$

Since x is a distance it cannot be negative so $x = 8$

We must prove that $x = 8$ does indeed give a minimum value of $T(x)$ using a nature table

x	1	8	10
$T'(x)$	-	0	+
<i>Shape</i>			

So $x = 8$ does give a minimum value of $T(x)$

$$\begin{aligned}T(8) &= 5\sqrt{36 + 8^2} + 4(20 - 8) \\ &= 98\end{aligned}$$

So, the minimum time possible is 98 tenths of a second, 9.8 seconds

2016 Paper 2 Question 7, (3) (6)

a) Length = $9x + 8y$

$$\text{Area} = 2y \cdot 3x = 108$$

$$6xy = 108$$

$$y = \frac{18}{x}$$

Substituting $y = \frac{18}{x}$ into Length gives

$$\begin{aligned}\text{Length} &= 9x + 8 \cdot \frac{18}{x} \\ &= 9x + \frac{144}{x}\end{aligned}$$

$$\text{So, } L(x) = 9x + \frac{144}{x}$$

b) Minimum values of $L(x)$ occur where $L'(x) = 0$

$$L(x) = 9x + \frac{144}{x} = 9x + 144x^{-1}$$

$$\begin{aligned}L'(x) &= 9 - 144x^{-2} \\ &= 9 - \frac{144}{x^2}\end{aligned}$$

Let $L'(x) = 0$ to give

$$9 - \frac{144}{x^2} = 0$$

$$9x^2 - 144 = 0$$




$$x^2 - 16 = 0$$

$$(x - 4)(x + 4) = 0$$

$$x = -4, x = 4$$

Since x is a length it cannot be negative so $x = 4$

We have to show that $x = 4$ gives a minimum value of $L(x)$

x	1	4	10
$L'(x)$	-	0	+
<i>Shape</i>			

So, $x = 4$ gives a minimum value of $L(x)$