Higher Mathematics

Exponential & Logarithm - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 20, (2)

The equation of the line is $\log_3 y = 2x$ since gradient $= 2$ and y-intercept $= 0$

$\log_3 y = 2x$

Rewriting as an exponential gives

$y = 3^{2x}$

2013 Paper 2 Question 5, (4)

$$\log_5(3-2x) + \log_5(2+x) = 1$$
$$\log_5\left[(3-2x)(2+x)\right] = 1$$
$$5^1 = (3-2x)(2+x)$$
$$5 = 6 - x - 2x^2$$
$$2x^2 + x - 1 = 0$$
$$(2x-1)(x+1) = 0$$
$$x = -1, x = \frac{1}{2}$$

2013 Paper 2 Question 9, (4) (3)

a) $p_t = p_0 e^{-kt}$

Since the concentration has halved $p_t = \frac{p_0}{2}$

$$\frac{p_0}{2} = p_0 e^{-25k}$$
$$0.5 = e^{-25k}$$
$$\log_e 0.5 = \log_e e^{-25k}$$
$$\log_e 0.5 = -25k$$

$$k = \frac{\log_e 0.5}{-25}$$

$k = 0.028$ to 2 significant figures

b) $p_t = p_0 e^{-kt}$

Let $t = 80$ and $k = 0.028$. Substitute to give

$p_t = p_0 e^{-0.028 \, x \, 80}$

$p_t = p_0 e^{-2.24}$

$p_t = 0.1065 \, p_0$

Rounding gives

$p_t = 0.11 \, p_0$

So, p_t is 11% of p_0

Therefore, the concentration has decreased by 89%

2014 Paper 1 Question 3, (2)

$$\log_4 12 - \log_4 x = \log_4 6$$

$$\log_4 12 - \log_4 6 = \log_4 x$$

$$\log_4 \frac{12}{6} = \log_4 x$$

$$\log_4 2 = \log_4 x$$

$$x = 2$$

2014 Paper 1 Question 20, (2)

$$2 - \log_5 \frac{1}{25} = 2 - (\log_5 1 - \log_5 25) = 2 - 0 + 2 = 4$$

2014 Paper 1 Question 24, (5)

$y = ka^x$

$\log_9 y = \log_9(ka^x)$

$\log_9 y = \log_9 k + \log_9 a^x$

$\log_9 y = \log_9 k + x\log_9 a$

Rewrite in the format of a straight line

$\log_9 y = (\log_9 a)x + \log_9 k$

Consider the line shown in the graph

$m = \dfrac{5-2}{6-0} = \dfrac{1}{2}$

Y intercept $= 2$

So, by inspection we have

$\log_9 a = \dfrac{1}{2}$ $\qquad$ $\log_9 k = 2$

$a = 9^{\frac{1}{2}} = 3$ $\qquad$ $k = 9^2 = 81$

2015 Paper 1 Question 6, (3)

$$\log_6 12 + \frac{1}{3}\log_6 27 = \log_6 12 + \log_6 27^{\frac{1}{3}}$$

$$= \log_6 12 + \log_6 3$$

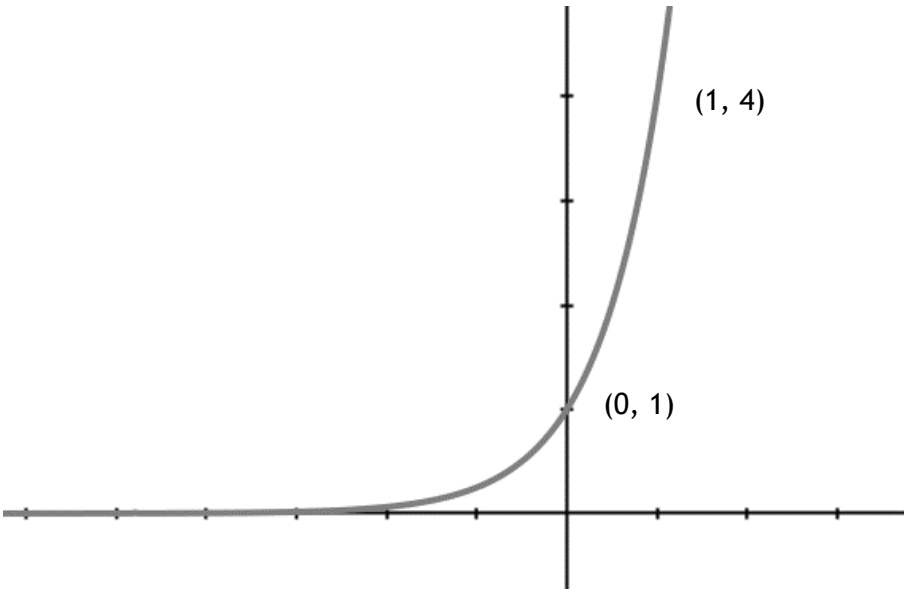
$$= \log_6 36$$

$$= 2$$

2016 Paper 1 Question 10, (2)

The point (1, 0) on the graph of $f(x)$ is transformed to (0, 1) on the graph of $f^{-1}(x)$

The point (4, 1) on the graph of $f(x)$ is transformed to (1, 4) on the graph of $f^{-1}(x)$



2016 Paper 1 Question 14, (1) (5)

a) $\log_5 25 = 2$ since $5^2 = 25$

b) $\log_4 x + \log_4(x-6) = \log_5 25$

$$\log_4 x + \log_4(x-6) = 2$$

$$\log_4 x(x-6) = 2$$

Rewriting as an exponential gives

$$x(x-6) = 4^2$$

$$x^2 - 6x = 16$$

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

$$x = -2, x = 8$$

2016 Paper 2 Question 6, (1) (4)

a) $B(t) = 200e^{0.107t}$

At the start $t = 0$ giving $B(0) = 200 \bullet e^0 = 200$

b) When the number doubles there are 400 bacteria

Let $B(t) = 400$

$$400 = 200e^{0.107t}$$

$$2 = e^{0.107t}$$

$$\log_e 2 = \log_e e^{0.107t}$$

$$\log_e 2 = 0.107t$$

$$t = \frac{\log_e 2}{0.107} = 6.48 \text{ hours}$$

2017 Paper 1 Question 12, (3)

$$\log_a 36 - \log_a 4 = \frac{1}{2}$$

$$\log_a\left(\frac{36}{4}\right) = \frac{1}{2}$$

$$\log_a 9 = \frac{1}{2}$$

Rewriting as an exponential to give

$$a^{\frac{1}{2}} = 9$$

$$a = 9^2 = 81$$

2017 Paper 2 Question 9, (5)

$$y = kx^n$$

$$\log_2 y = \log_2 kx^n$$

$$\log_2 y = \log_2 k + \log_2 x^n$$

$$\log_2 y = \log_2 k + n\log_2 x$$

Reorder to give

$$\log_2 y = n\log_2 x + \log_2 k$$

n is the line gradient $= \dfrac{3-0}{0-(-12)} = \dfrac{1}{4}$

$\log_2 k$ is the y intercept

$$\log_2 k = 3$$

$$k = 2^3 = 8$$