Higher Mathematics

Differentiation - **Solutions** - 2013-2017

Marks are indicated in brackets after each question number

2013 Paper 1 Question 2, (2)

$$y = x^2 - 4x + 7$$

$$\frac{dy}{dx} = 2x - 4$$

Gradient of tangent at (5, 12) $= 2 \bullet 5 - 4 = 6$

Using $y - b = m(x - a)$ gives

$$y - 12 = 6(x - 5)$$

$$y - 12 = 6x - 30$$

$$y = 6x - 28$$

2013 Paper 1 Question 18, (2)

$$y = \sin(x^2 - 3)$$

$$\frac{dy}{dx} = \cos(x^2 - 3) \bullet 2x$$

$$= 2x\cos(x^2 - 3)$$

2014 Paper 1 Question 8, (2)

Let $f(x) = (4 - 9x^4)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}} \bullet (-36x^3)$$

$$= -18x^3(4 - 9x^4)^{\frac{1}{2}}$$

2014 Paper 1 Question 9, (2)

Note there are various ways to solve this problem. This approach uses the derivative to find max values

Let $f(x) = 5\sin 2x + 5\sqrt{3}\cos 2x$

$f'(x) = 10\cos 2x - 10\sin 2x$

For maximum values $f'(x) = 0$

$10\cos 2x - 10\sin 2x = 0$

$\cos 2x - \sin 2x = 0$

$\cos 2x = \sin 2x$

$1 = \dfrac{\sin 2x}{\cos 2x} = \tan 2x$

From exact values $2x = \dfrac{\pi}{6}$

So, the maximum value occurs where $2x = \dfrac{\pi}{6}$

Substituting $2x = \dfrac{\pi}{6}$ gives

$f(x) = 5\sin\dfrac{\pi}{6} + 5\sqrt{3}\cos\dfrac{\pi}{6} = \dfrac{5}{2} + \dfrac{15}{2} = 10$

2014 Paper 1 Question 21, (6) (2)

a) $y = 3x^2 - x^3$

$\dfrac{dy}{dx} = 6x - 3x^2$

For stationary points $\dfrac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, x = 2$

When $x = 0, y = 0$

When $x = 2, y = 4$

So, stationary points are $(0, 0)$ $(2, 4)$

x	-1	0	1	2	5
$f'(x)$	+	0	−	0	+
Shape	╲	___	╱	‾‾‾	╲

So, $(0, 0)$ is a minimum turning point and $(2, 4)$ is a maximum turning point

b) For the y intercept $x = 0$ giving

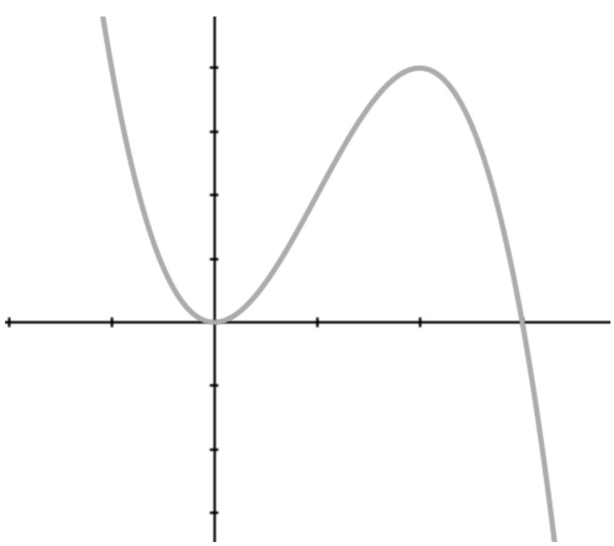
$$y = 3 \bullet 0^2 - 0^3 = 0$$

For the x intercept $y = 0$ giving

$$0 = 3x^2 - x^3$$

$$0 = x^2(3 - x)$$

$$x = 0, x = 3$$



2014 Paper 2 Question 2, (4)

$y = x^4 - 2x^3 + 5$

$\frac{dy}{dx} = 4x^3 - 6x^2$

When $x = 2, \frac{dy}{dx} = 32 - 24 = 8$

So, the gradient of the tangent at $x = 2$ is 8

When $x = 2, y = 2^4 - 2 \bullet 2^3 + 5 = 5$

Using $y - b = m(x - a)$ with (2, 5) gives

$y - 5 = 8(x - 2)$

Rearranging gives

$y = 8x - 11$

2015 Paper 1 Question 2, (4)

$y = 2x^3 + 3$

$\frac{dy}{dx} = 6x^2$

When $x = -2, \frac{dy}{dx} = 6 \bullet (-2)^2 = 24$

So, gradient of tangent at $x = -2$ is 24

When $x = -2, y = 2 \bullet (-2)^3 + 3 = -13$

So, $(-2, -13)$ lies on the curve

Using $y - b = m(x - a)$ we have

$y - (-13) = -2(x - (-13))$

$y + 13 = -2x - 26$

$y = -2x - 39$

2015 Paper 1 Question 7, (4)

$$f(x) = \sqrt{x}(3x - \frac{2}{x\sqrt{x}})$$

$$= x^{\frac{1}{2}}(3x - \frac{2}{x \bullet x^{\frac{1}{2}}})$$

$$= x^{\frac{1}{2}}(3x - \frac{2}{x^{\frac{3}{2}}})$$

$$= x^{\frac{1}{2}}(3x - 2x^{-\frac{3}{2}})$$

$$= 3x^{\frac{3}{2}} - 2x^{-1}$$

$$f'(x) = \frac{9}{2}x^{\frac{1}{2}} + 2x^{-2} = \frac{9}{2}x^{\frac{1}{2}} + \frac{2}{x^2}$$

$$f'(4) = \left(\frac{9}{2} \bullet 2\right) + \frac{2}{16} = 9 + \frac{1}{8} = \frac{73}{8}$$

2016 Paper 1 Question 2, (3)

$$y = 12x^3 + 8\sqrt{x}$$

$$y = 12x^3 + 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$$

2016 Paper 1 Question 9, (4) (2)

a) $f(x) = x^3 + 3x^2 - 24x$

$f'(x) = 3x^2 + 6x - 24$

For stationary points $f'(x) = 0$

Let $f'(x) = 0$

$$3x^2 + 6x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2)=0$$

$$x=-4, x=2$$

b)

x	-10	-4	1	2	10
$f'(x)$	$+$	0	$-$	0	$+$
$Shape$	/	—	\	—	/

So, f is strictly increasing for $x<-4$ and for $x>2$

So, $-4>x>2$

<u>2016 Paper 2 Question 10, (2) (1)</u>

a) $y=(x^2+7)^{\frac{1}{2}}$

$$\frac{dy}{dx}=\frac{1}{2}(x^2+7)^{-\frac{1}{2}}\bullet 2x$$

$$=x(x^2+7)^{-\frac{1}{2}}$$

$$=\frac{x}{(x^2+7)^{\frac{1}{2}}}$$

$$=\frac{x}{\sqrt{x^2+7}}$$

b) $\int \frac{4x}{\sqrt{x^2+7}}\,dx$

$$=4\int \frac{x}{\sqrt{x^2+7}}\,dx$$

$=4(x^2+7)^{\frac{1}{2}}$ (using the result from a)

2017 Paper 1 Question 3, (2)

$y = (4x - 1)^{12}$

$\frac{dy}{dx} = 12(4x - 1)^{11} \bullet 4$

$= 48(4x - 1)^{11}$

2017 Paper 1 Question 8, (3)

$d(t) = \frac{1}{2t} = \frac{t^{-1}}{2} = \frac{1}{2}t^{-1}$

$d'(t) = -\frac{1}{2}t^{-2} = -\frac{1}{2t^2}$

$d'(5) = -\frac{1}{2 \bullet 5^2} = -\frac{1}{50}$

So, the rate of change of $d(t)$ when $t = 5$ is $-\frac{1}{50}$

2017 Paper 2 Question 7, (4) (3)

a) $y = 6x - 2\sqrt{x^3}$

$y = 6x - 2x^{\frac{3}{2}}$

$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$

For stationary points $\frac{dy}{dx} = 0$

$6 - 3x^{\frac{1}{2}} = 0$

$6 = 3x^{\frac{1}{2}}$

$2 = x^{\frac{1}{2}}$

Squaring both sides gives

$4 = x$

b) When $x = 4, y = 8$

When $x = 1, y = 4$

When $x = 9, y = 0$

So, the maximum value of y on $1 \leq x \leq 9$ is 8

And the minimum value is 0