Higher Mathematics

Composite Functions - **Solutions** - 2013-2017

2013 Paper 1 Question 1, (2)

$$g\left(f(x)\right) = g\left(x^2+1\right) = 3\left(x^2+1\right) - 4$$
$$= 3x^2 + 3 - 4$$
$$= 3x^2 - 1$$

2014 Paper 2 Question 3, (2)

a) $f\left(g(x)\right) = f(x+3) = (x+3)(x+3-1) + q$
$$= (x+3)(x+2) + q$$
$$= x^2 + 5x + 6 + q$$

b) $f\left(g(x)\right) = x^2 + 5x + 6 + q = x^2 + 5x + (6+q)$

$a = 1, b = 5, c = 6 + q$

For equal roots $b^2 - 4ac = 0$

$$25 - 4 \bullet 1 \bullet \left(6+q\right) = 0$$
$$25 - 24 - 4q = 0$$
$$q = \frac{1}{4}$$

2015 Paper 1 Question 5, (2) (1)

a) $g(x) = 6 - 2x$

$y = 6 - 2x$

Interchange x y to give

$x = 6 - 2y$

Solve for y

$$y = 3 - \frac{1}{2}x$$

So, $g^{-1}(x) = 3 - \frac{1}{2}x$

b) $g\left(g^{-1}(x)\right) = x$ since g and g^{-1} are inverses of each other

<u>2015 Paper 2 Question 2, (2) (3) (2)</u>

a) $f\left(g(x)\right) = f((1+x)(3-x)+2)$

$$= 10 + (1+x)(3-x) + 2$$

Simplifying gives

$$f\left(g(x)\right) = 15 + 2x - x^2$$

b) $f\left(g(x)\right) = -x^2 + 2x + 15$

$$= -\left(x^2 - 2x\right) + 15$$

Completing the square of the inside of the bracket

$$= -\left[(x-1)^2 - 1\right] + 15$$

$$= -(x-1)^2 + 16$$

c) $h(x) = \dfrac{1}{f(g(x))} = \dfrac{1}{-(x-1)^2 + 16}$

Restrictions on the domain of $h(x)$ occur where $-(x-1)^2 + 16 = 0$

Let $-(x-1)^2 + 16 = 0$

$$(x-1)^2 = 16$$

$$x - 1 = 4 \qquad x - 1 = -4$$

$$x = 5 \qquad x = -3$$

So, $x = -3, x = 5$ cannot be in the domain of $h(x)$

2016 Paper 1 Question 12, (2) (3)

a) $h(x) = f\left(g(x)\right) = f(3-x) = 2(3-x)^2 - 4(3-x) + 5 = 2x^2 - 8x + 11$

b) $h(x) = 2x^2 - 8x + 11 = 2\left(x^2 - 4x\right) + 11$

$$= 2\left[(x-2)^2 - 4\right] + 11$$

$$= 2(x-2)^2 - 8 + 11$$

$$= 2(x-2)^2 + 3$$

2017 Paper 1 Question 1, (1) (2)

a) $f\left(g(x)\right) = f(2\cos x) = 5 \bullet 2\cos x = 10\cos x$

$f\left(g(0)\right) = 10 \bullet \cos 0 = 10$

b) $g\left(f(x)\right) = g\left(5x\right) = 2\cos(5x)$